RAND

New Capabilities for Strategic Mobility Analysis Using Mathematical Programming

Michael G. Mattock, John F. Schank
James P. Stucker, Jeff Rothenberg

National Defense Research Institute

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PREFACE

The Joint Staff and the Office of the Secretary of Defense consider long-range force structure issues. Such issues have become more pressing following the end of the Cold War, because a wider range of scenarios in diverse parts of the world complicates logistics planning. To aid in resolving a variety of logistics-planning issues, RAND conducted a four-task project entitled "Achieving Maximum Effectiveness from Available Joint/Combined Logistics Resources." The research was sponsored by the Logistics Directorate of the Joint Staff (JS/J-4) and was conducted in the Acquisition and Technology Policy Center of RAND's National Defense Research Institute, a federally funded research and development center sponsored by the Office of the Secretary of Defense, the Joint Staff, and the defense agencies.

The first two tasks were to survey the needs and opportunities for responsive logistics and operations command and communication, and to conceive and evaluate enhancements for conventional ammunition. They are documented in the following reports:

> S. C. Moore, J. Stucker, and J. Schank, *Wartime Roles and Capabilities for the Unified Logistic Staffs*, RAND, R-3716-JCS, February 1989.
>
> J. Schank, et al., *Enhancing Joint Capabilities in Theater Ammunition Management*, RAND, R-3789-JS, 1991.

The objective of the third task was to understand the capabilities of the major computerized models and databases used for analyzing strategic mobility questions, to survey the various uses of strategic mobility models, to evaluate the attributes and limitations of the major existing models, and to determine whether another type of computer model would serve the directorate's needs better than does its current model. This task is documented in the following report:

> J. Schank, et al., *A Review of Strategic Mobility Models and Analysis*, RAND, R-3926, 1991.

In the third task, we found several shortcomings with existing mobility models, especially in their application to problems of transportation resource requirements. We recommended that the JS/J-4 pursue the development of new modeling capabilities and suggested that mathematical programming and a new knowledge-based modeling environment being developed under Advanced Research Projects Agency sponsorship would be promising technologies for these new analysis capabilities.

This report documents the results of the fourth task, which was aimed at developing and demonstrating a mathematical programming prototype designed specifically for transportation requirements analysis. Another report describing results of this phase of the research is

> J. Schank, et al., *New Capabilities for Strategic Mobility Analysis: Executive Summary*, RAND, MR-294-JS, 1994.

This document should interest both policymakers and analysts in the strategic mobility community, especially those at the Office of the Secretary of Defense for Program Analysis and Evaluation and at the United States Transportation Command.

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SUMMARY

BACKGROUND

Strategic mobility issues have long concerned various elements of the Department of Defense (DoD). Theater commanders and the United States Transportation Command (USTRANSCOM) are interested in the ability of the military transportation system (i.e., the collection of aircraft, ships, airfields, and seaports used to move cargoes; it is also referred to as the *mobility system)* to support contingency plans; they and the services determine the best use of available assets to respond to emergencies. The Joint Staff and the Office of the Secretary of Defense consider long-range force structure issues. Such issues have become more pressing following the end of the Cold War. Where once strategic mobility analysts could focus on a large-scale NATO-versus-Warsaw Pact war, now they must address a wide range of scenarios in diverse parts of the world, which significantly complicates transportation planning.

Computer models have played an important role in analyzing strategic mobility requirements. The transportation system is so large and complex that it would take far too long to plan or evaluate movements of any size without them. Models currently in use are primarily deterministic simulations; they take as inputs information about cargoes, planes, ships, and ports and produce estimates about when cargoes could be delivered. These models were developed to answer questions about the ability of the current transportation fleet to meet required delivery dates, and they perform this function reasonably well.

However, models currently in use do not deal well with questions about how many assets would be required to deliver cargoes by a specific date, because they require the information being sought as part of the input. That is, the analyst must define the assets available before the model will run. To find a "best" solution, the analyst must vary the inputs on a trial-and-error basis, frequently running the models hundreds of times, searching for an answer. Not only is this process inefficient and time-consuming, but it also does not guarantee that the result will be optimum in any real sense of the word. It will simply represent the best solution from the many model runs.

PURPOSE

This drawback prompted RAND to recommend that the Joint Staff develop new models specifically to address transportation requirements issues. RAND identified two technologies as promising: knowledge-based modeling and mathematical programming modeling. This report describes the development and operating characteristics of a family of strategic mobility models using the mathematical programming approach. It also includes the formulas and computer programs so that others can verify, duplicate, or extend the model.

WHY MATHEMATICAL PROGRAMMING?

Models are clearly useful in analyzing large or complex problems. As useful as they are, most simulation models used in mobility analysis are informal. That is, most of them are written in programming languages and do not rest on formal mathematical or logical principles. Thus, although they may make complex issues easier to understand, it is still difficult to prove anything based on them. Mathematical models, on the other hand, contain equations, rest on demonstrated mathematical principles, and are subject to definitive proofs.

Mathematical programming (MP) is the general term that applies to a family of solution techniques for a wide range of problems. MP formulations offer a number of advantages for addressing transportation requirements questions. MP models

- directly provide optimal answers
- consider all possible combinations of inputs
- provide information on excess capacities in the system
- define the economic advantage of obtaining additional constrained resources.

These characteristics offer the mobility analyst a powerful tool. An MP model can define the optimal solution for the analyst, and, if the optimal solution is impractical, it will help examine trade-offs among the different components of the transportation system to find the "best" practical solution. Thus, the analyst can determine the optimal answer with various constraints (e.g., limited budget to acquire new assets) or by easing different system parameters (e.g., allowing some units to arrive later than planned).

DISADVANTAGES OF MP

All models have drawbacks, and MP models are no exception. Their primary drawback is that they can require an enormous number of calculations, which tends to double with each additional variable. That is, adding another port, cargo, or transportation asset will double the number of calculations required. This characteristic has, in part, been responsible for the delay in using MP models, which have been theoretically proven for decades. But until the recent advances in computers and solution algorithms, MP models have simply been impractical, because their calculations take too long. Even today's computers have difficulty solving large transportation problems. A moderate-sized problem could involve trillions of calculations and take days or even weeks to solve. In the prototype, we have attempted to identify and capture the level of detail necessary to produce acceptable solutions. We eliminate the detail that is extraneous or unrealistic (because of future uncertainties).

Partly because of the computational need to reduce the size of the MP formulation and partly because it is difficult to capture the intricate nature of the real world in analytic equations, MP models typically lack the detail necessary to fully represent the real world. Also, the real world does not act in an optimal fashion. Therefore, answers resulting from MP models often understate the true requirement. Simulation models, on the other hand, can capture real-world operations and represent more detail in their formulation. The two types of models can act in concert to provide a better analytical environment than either by itself; the output of an MP model can be used as input to a simulation to provide a reasonable starting point for the simulation. The output of the simulation can then be used to calibrate the factors and coefficients in the MP formulation.

USING AGGREGATION TO DEAL WITH LARGE TRANSPORTATION PROBLEMS

We attempted to use aggregation—for example, by combining cargoes of similar characteristics—as a way of reducing the number of calculations needed. We adopted the general principle that any modification of the problem that did not change a binding constraint (i.e., any of the factors that drive the solution) would not change the answer. Running the model in both aggregated and disaggregated modes, we then determined which aggregations had no effect on the outcome and which did. We determined that we could aggregate movement requirements that had identical ports (departure and arrival), cargo types, and loading dates. We could also aggregate ports that were not bottlenecks. And it was possible to determine fleet requirements by considering only peak periods. To illustrate the effect of the aggregation techniques, we applied them to a standard data set provided by the Advanced Research Projects Agency (ARPA). In its original formulation, the data set had more than 10 million variables. Aggregation allowed us to reduce that number to about 4400 and still obtain identical results.

THE MP MODEL

We did not develop a single model to address the Joint Staff's mobility concerns. Rather, we developed a family of models, each of which addresses one of the tradeoffs inherent in the overall transportation system. The basic model is a cost-minimization model, which views the transportation system as being composed of a number of objects that must move through a system on different transportation assets. Both the cargoes and the transportation assets have various characteristics. The model recognizes four types of cargoes: passengers, bulk, oversized, and outsized. Each cargo has two timing considerations—when it is available to load and when its delivery is required—and two locations—port of embarkation (POE) and port of debarkation (POD).

Transportation assets include aircraft (C-141, C-5, etc.) and numerous different kinds of ships (RoRo, breakbulk, etc.). Each asset has distinctive characteristics. Not all assets can carry all cargo. C-141s, for example, cannot carry outsized cargo. The model distinguishes between assets currently in the fleet and those that can be purchased or leased (e.g., Civil Reserve Airlift Fleet, Ready Reserve Fleet). New assets, such as the C-17 or fast ships, can be added. Each newly acquired asset has a cost associated with its acquisition and use.

The transportation system is composed of a number of POEs and PODs, each pair representing a channel through which cargo must transit. The ports have geographical locations (and thus a distance from each other) and a capacity measured in tons per day of throughput capacity.

The model is formulated to minimize the total cost of acquiring (buying and operating or leasing) assets above those currently in the fleet. The model delivers all cargoes by their required delivery date (RDD). It develops a time window for each cargo (one for each type of asset that can carry it) that defines the earliest and latest a cargo can leave a port and still meet the delivery date. The model first calculates the number of vehicles in a particular channel on a particular day, then sums over all channels to find out how many assets are used each day. The mathematical formulation of the model causes it to choose the least costly alternative that meets all constraints.

TRADE-OFFS

However, the least costly alternative as determined by the model may not be the most practical or even feasible in the real world. For example, the model might provide a solution that exceeds the available budget. The other models in the family enable the analyst to impose various constraints and derive more-practical alternatives. The approach involves modifying the basic cost-minimization model by adding constraints and variables and altering its focus. An attractive aspect of this approach is that it involves changing only a few equations.

For example, to determine the effect of a trade-off between cost and allowing some cargo to arrive later than the RDD, we simply constrain cost to be less than a given amount, add a variable to the model that allows it to ship cargo late, and change the objective to be one of minimizing lateness rather than cost. The model will tell the analyst the number of late ton-days the cost constraint will cause.[1] The models will determine other trade-offs as well. They can also examine the effect of shipping some cargo early or prepositioning cargo.

The models can also analyze multiple scenarios. This aspect of the models enables analysts to determine which set of transportation assets is robust across a variety of

[1]A *ton-day* is a measure of timeliness (early or late) and is simply the number of tons of cargo multiplied by the number of days.

scenarios. That is, the models will define a transportation fleet that may not be optimum for any given scenario but will satisfy the demands of a wide range of scenarios.

INTENDED USE OF MP MODELS

The Joint Staff participates in all aspects of strategic mobility analysis. However, their primary strategic mobility role, demanding the majority of their analytic time and effort, centers on balancing troops, transport, and costs. Joint Staff analysts seek to identify the "best" mix of airlift, sealift, and prepositioning for use in responding to specified contingencies in what are often termed *illustrative planning scenarios.* This volume documents the mathematical programming formulations we developed to enable the Joint Staff Logistics Directorate (J-4) to analyze those trade-offs between transport, readiness, employment times, and prepositioning. The analyses investigate preferred packages of military resources for future years. They are not concerned with short-term efficiencies or with the timing of purchases. Instead, they optimize the long-term structure of the military transportation system. To do so, they take the current and projected assets and operations of military transport as given. That is, the current force and its peacetime operations form the base case, and the models (1) investigate the ability of base-case assets to handle contingency operations that may (or may not) be required on top of or in place of some interval (probably several months) of the base-case operations or (2) determine the augmentation that must be made available to handle the contingencies. This does not imply that the models minimize the costs expended during future contingency operation.

The contingencies may never occur. If any of them do, the DoD, the Congress, and the nation will, at that time, decide how much money (and lives, industrial capacity, etc.) the nation is willing to spend to deal with them. Our analyses reported here investigate, instead, how best to provide capabilities that can be used in whatever contingencies do occur and how to provide those capabilities over a 30-year planning horizon at the least cost.

The analyses we describe take current or near-term projections of the military transport system and its operations as given. They investigate the capability of the system in selected scenarios. Then they investigate the effects of changes and additions to the transport system and identify the changes that will improve present capabilities to meet the scenario or scenarios for the least expenditure of resources. All incremental resources are costed over the full 30-year horizon, considering all currently expected acquisition, operation, and upkeep expenditures. As noted above, expected expenditures on current assets and operations are not considered. If base-case assets and/or their operations are assumed to be replaced by incremental assets, then only the net costs of the incremental assets are considered. The costs of incremental assets and their operations were developed at RAND in coordination with a study of the future DoD distribution system.

The focus throughout this document remains on transportation. When our model explores interactions between transport and other factors, such as required delivery dates, availability dates, or prepositioning options, our examples typically minimize only incremental transportation costs. In situations involving trade-offs between transport and those other variables, however, the runs output information on *shadow prices* (i.e., sensitivity measures that define the economic advantage of having additional capacity) for those variables, information that can be used to determine whether such trade-offs are in fact relevant. Our formulations permit broader minimizations when costs of all *variables* (items of interest) are considered, but such analyses are not demonstrated because such cost information is currently not available.[2]

We might conceive of situations in which the country could procure the capabilities needed for a contingency only if and when a contingency occurs. For example, if a contingency requires only the transport of modest amounts of troops and materiel along established and well-defended air or rail routes, the military might be able simply to lease those capabilities from a commercial airline or package-delivery company when it needs them. More often, however, the military requirement will differ significantly from the civil capability and will be substantially more stressful and dangerous. So the military must expect to deal with the contingency by having its organizations procure specialized equipment and personnel, then holding them ready and training the personnel for specialized military operations. Note also that the current assets are typically expected to operate over the entire period of the contingency. And if they need to be modified or upgraded, the costs of doing so are included in their baseline and not considered in our optimizations. Costs of that type are netted out only when we investigate the replacement of current assets by incrementally procured assets.

[2]For discussion of the Joint Staff roles in strategic mobility analysis, see Schank, et al., 1991b.

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Many individuals provided substantial help in understanding the various uses of strategic mobility models and the characteristics and procedures of the major models we investigated. Col Andrew J. McIntyre (USAF) first formulated the original ideas and objectives for the study. Maj Gen Gary Mears (USAF) provided overall guidance to our research. On the Joint Staff, Col William Smiley (USAF, Retired), CDR Kevin Kelly (USN), Maj Gary Arnett (USAF), and Thomas Currier all shared their time and expertise on mobility analysis. Special thanks must go to COL Richard Strand (USA), who was our guide, critic, and facilitator during Task 3 and at the start of Task 4. We also appreciate the help of LTC Joe Selman (USA) who stepped in for COL Strand and guided the Task 4 analysis to completion, and LCDR Robert Drash (USN), who was a perceptive critic and has contributed greatly to the evolution of the methods described in this report.

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ACRONYMS AND ABBREVIATIONS

ALD	Available to load date
AMC	Air Mobility Command
ARPA	Advanced Research Projects Agency
CRAF	Civil Reserve Airlift Fleet
GAMS	General Algebraic Modeling System
J-4	Joint Staff Logistics Directorate
JS	Joint Staff
LAD	Latest arrival date
LCC	Life-cycle cost
LP	Linear programming
LRWC	Long-range, wide-bodied cargo [aircraft]
LRWP	Long-range, wide-bodied passenger [aircraft]
MODES	Mode Optimization and Delivery Estimate System
MP	Mathematical programming
MSC	Military Sealift Command
MTMC	Military Traffic Management Command
OSD (PA&E)	Office of the Secretary of Defense (Program, Analysis, and Evaluation)
POD	Port of debarkation
POE	Port of embarkation
RDD	Required delivery date
RRF	Ready Reserve Force
SCOPE	System for Closure Optimization Planning and Evaluation
USTRANSCOM	U.S. Transportation Command

INTRODUCTION

BACKGROUND

Strategic mobility involves moving forces (units, support, and resupply) from their home station to an employment location. Numerous defense organizations address strategic mobility issues, and their analyses generally fall into three categories: The theater commanders, the United States Transportation Command (USTRANSCOM), and the transportation providers (the Air Mobility Command [AMC], the Military Sealift Command [MSC], and the Military Traffic Management Command [MTMC]) consider the mobility aspects of **deliberate planning**, that is, how the allocated transportation assets can best support the theater commanders' plans. Those same organizations, in coordination with the services, address **crisis action and execution planning** issues, or determining the best use of available transportation assets to respond to emergencies. The Joint Staff, Office of the Secretary of Defense (Program, Analysis, and Evaluation), and the service headquarters consider questions of **long-range force structure requirements**, that is, issues surrounding the type and quantity of aircraft, ships, and prepositioned assets that support strategic mobility. The research described here addresses the third category and describes how a modeling approach can assist analysts in dealing with such issues.

Recent changes in the world political climate have made strategic mobility issues even more complex and difficult to address. As a result of the withdrawal of former Soviet forces from Eastern Europe and the collapse of the Soviet empire, a portion of U.S. forces have been withdrawn from bases in Europe and the U.S. defense budget has been reduced. The lessening military tension following the collapse of the Soviet Union is counterbalanced by the increasing unrest and friction in the Third World and elements of the former Soviet Union. No longer do strategic mobility analyses focus only on the rapid deployment of forces to Europe. Now, analysts must address questions regarding where forces may have to be sent, what forces will be required, and whether the ultimate location has an infrastructure to support airlift and sealift operations.

Studies of the third category of strategic mobility issues focus on the resources required by the mobility system—that is, the number and types of transportation assets needed to deliver a set of cargoes by their specified dates. However, such studies must also consider the trade-offs in the overall transportation system, such as relaxing delivery dates or cargo availability dates, or prepositioning cargoes. Increases or decreases in any of these parameters can affect transportation requirements.

To address these long-range resource issues, analysts frequently employ mobility models. The models currently used by organizations involved in strategic mobility analysis are primarily *deterministic simulations*: They accept information about the cargoes to be moved, the ships and planes available to move cargoes, and the overall transportation system, and they produce estimates of when cargoes can be delivered. These models order cargoes according to some ranking scheme, select them one at a time, and route them through the system according to rules they contain. These models were built to address questions concerning the capabilities of the existing transportation fleet to meet desired force-delivery dates. Although all existing models have some limitations, they appear to perform such capability assessments adequately. However, questions involving how many transportation assets are required to deliver a set of cargoes by specified (closure) dates are difficult to answer using existing models.

They are difficult to answer because the information that is unknown—the number of aircraft and ships—is required as input. Therefore, analysts must vary input values on a trial-and-error basis, hoping to find the combination that achieves the desired closures. It is not unusual for analysts to run an existing model hundreds of times in their search for an answer. And, even if a combination of assets that meets desired objectives emerges, there is no guarantee that the resulting transportation fleet is "optimal" in any way.

Furthermore, transportation requirements issues go beyond the question of how many of what types of assets are needed. The Joint Staff must also examine a number of related questions, including the following:

- What assets should be prepositioned and where?
- When should specific units be available to load?
- When should reserve mobility assets[1] be activated and how many?
- How do answers differ for different scenarios?

Existing models do not address these questions well, either. Because of the difficulty in answering this range of questions using existing models, we recommended to the Joint Staff that new models be developed, models specifically designed to address the questions associated with transportation requirements issues. We further identified two technologies, mathematical programming (MP) and a new knowledge-based modeling environment being developed under Advanced Research Projects Agency (ARPA) sponsorship that appeared to offer promise for implementing these new models.

[1] *Reserve mobility assets* include the aircraft in the Civil Reserve Aircraft Fleet (CRAF) and the ships in the Ready Reserve Fleet (RRF).

RESEARCH OBJECTIVES

In this research, we focused on developing a mathematical programming prototype that would fulfill three objectives:

- To demonstrate the ability of mathematical programming to address directly the question of the least-cost set of transportation assets required to deliver all cargoes on time
- To develop alternative formulations of the solution technique to address the various trade-offs in the overall transportation system
- To understand the effect on the optimal answer of aggregating different aspects of the system representation and of the data.

PURPOSE AND ORGANIZATION OF THIS DOCUMENT

The report has two intended audiences and a separate purpose for each. First, it tells policymakers why the current suite of mobility models does not provide satisfactory answers to questions about long-range force structure, describes the mathematical programming approach, and provides two examples—one simple and one complex—to illustrate how the mathematical programming models work. Second, the document provides analysts with the formulas and computer codes necessary to verify the model and replicate it for their own use. In general, the technical material, such as mathematical formulations and calculations, appears at the end of chapters and can be bypassed by those readers interested only in the policy implications. Chapter Two provides an overview of mathematical programming, describes its historical applications, and discusses its advantages and disadvantages. Chapter Three describes how we applied mathematical programming to the military transportation system and provides a simple illustration. It also provides the mathematical formulas. Chapter Four describes how we dealt with one of the disadvantages of mathematical programming: its tendency to significantly increase the number of calculations required as the model considers additional variables. Chapter Five describes how the model makes trade-offs among components of the transportation system. Chapter Six contains an extended illustration of an analysis. Finally, Chapter Seven provides results of the modeling effort, describes the current status of the models, and makes recommendations. There are two appendices, which are intended for technical readers. Appendix A contains computer codes and other material needed to enable others to run the model. Appendix B illustrates the use of shadow prices for sensitivity analysis.

Chapter Two

MOTIVATION FOR THE MP APPROACH

This chapter provides a brief overview of the mathematical programming approach to solving transportation problems, describes previous attempts at applying MP techniques to this class of problems, lists some of the advantages of this approach, and describes difficulties that arise during model formulation and solution.

OVERVIEW

We build models of real-world phenomena or systems to improve our understanding of them. In general, the real world is complex and difficult to observe and understand; it is all but impossible to analyze directly or to prove anything about. Any model is necessarily simpler than the real world, which makes it easier to observe the model than to observe the real system under analysis. In principle, this simplicity makes it easier to understand the model than to understand the real system it models (although this is not always the case).

However, most simulation models typically used in mobility analysis are very informal models; they consist of programs in traditional programming languages, which have no formal mathematical or logical basis. Therefore, although simulation models may be easier to understand than the real system they model, it is still very difficult to analyze them or to prove anything about them.

Although difficult to analyze, simulation models are useful ways to represent complex systems that are not readily amenable to mathematical formulations, such as combat. Simulations play a vital role in many types of analyses and have been the primary strategic mobility analysis tool for over two decades. Simulation models can be used in conjunction with mathematical models, each taking advantage of the other's strengths, to provide a more complete and effective modeling environment than either type of model can provide independently.

In contrast, mathematical programming models are based on sound mathematical formalisms; i.e., they are capable of mathematical proof. This characteristic makes it possible to understand and analyze such models using formal mathematical analysis techniques, which enable proofs to be made about such models. This formality distinguishes mathematical programming from traditional simulation and provides the motivation for our prototyping efforts in this direction.

Mathematical programming is the general term applied to a family of solution techniques that determine optimal solutions for a wide range of problems. Specific MP formulations are influenced by the characteristics of the problem and solution space (the allowable values), and include linear programming, nonlinear programming, integer programming, goal programming, and other variations.

HISTORICAL APPLICATION OF MP TO TRANSPORTATION PROBLEMS

Linear programming (LP) is the typical solution technique for "simple" transportation problems. Such problems have supplies of a commodity at various starting points and demands for that commodity at various final destinations. Each route (a supply-demand link) has an associated cost per unit shipped. The objective is to minimize total shipping costs while satisfying all demands.

Strategic mobility problems, and real-world problems in general, are much more complex than simple textbook transportation problems. The major source of complexity is the addition of a time dimension—cargoes have times when they are available for loading and when they are required at their destination. Also, different types of transportation assets are available, each with a specified speed, capacity, and cost. The cargoes themselves are not homogeneous but have different dimensions (length, width, depth) and characteristics, some of which preclude shipping them on certain types of transport assets. There are, therefore, different cost factors for shipping cargo over a route, one for each type of transportation asset. Taking time into account, transport assets can be used again. Finally, strategic mobility problems allow *multiporting*, the ability of a transport asset to pick up cargo at various supply points and deliver its shipments to various destinations.

Several attempts have been made in the past to apply MP solution techniques to strategic mobility problems. The first attempts were during the 1950s, the heyday of the development and application of linear programming. George Dantzig, the main force behind linear programming and the developer of the simplex solution technique, applied linear programming to various aspects of transportation problems.[1] These earliest attempts successfully solved narrow segments of the overall problem, but the limited hardware and software of the time precluded consideration of the overall transportation requirement and of the trade-offs that exist within the transportation system. The difficulty of solving large MP problems has plagued analysts over the years.

A more recent attempt at applying mathematical programming to strategic transportation problems was undertaken by the Georgia Institute of Technology for the Joint Deployment Agency in 1983. Their SCOPE (System for Closure Optimization Planning and Evaluation) model (also known as MODES, Mode Optimization and

[1]Dantzig and other RAND analysts published a series of Notes on Linear Programming in the 1950s that describe various applications and solution techniques of linear programming. Specific RAND documents in this series include RM-1328, *Minimizing the Number of Carriers to Meet a Fixed Schedule*, August 1954; RM-1369, *The Problem of Routing Aircraft—A Mathematical Solution*, September 1954; and RM-1833, *The Allocation of Aircraft to Routes—An Example of Linear Programming Under Uncertain Demand*, December 1956.

Delivery Estimate System)[2] used the linear programming transportation algorithm to allocate lift assets to channels and to schedule the movement of materiel to meet delivery requirements at the destinations, subject to the constraints imposed by the loading capacities at the ports of embarkation (POEs), the unloading capacities at the ports of debarkation (PODs), the available lift assets, and the initial location of the materiel. It modeled airlift and sealift as long as each lift asset had only a single POE and a single POD; it could not handle multiport tours.

SCOPE/MODES was unsuccessful for two related reasons. While the model was under development, the Joint Deployment Agency continually added requirements. These additional details caused the formulation to grow dramatically. This growth, in turn, overtaxed the rather limited computer capabilities of the time and the existing solution algorithms. It became more and more difficult to generate solutions to reasonably sized scenarios. What started out to be a fairly high-level (low-resolution) model with promise quickly became a low-level (high-resolution) model that could be formulated but not solved.

ADVANTAGES OFFERED BY MP FORMULATIONS

MP formulations offer four advantages over simulation models for analyzing transportation requirements questions. First, **MP models directly provide the optimal answer to the problem.** MP formulations maximize or minimize some of the decision variables (termed the *objective function*) subject to a set of system constraints. The answer may relate to the least-cost set of transport assets needed to deliver the cargoes when desired, or to the minimum amount of delay possible when transportation assets are limited, or to any other function of the decision variables. Analysts do not need to set up and interpret multiple runs of an MP model, as they must with current simulations, unless they are interested in the effects of changing various system parameters or factors.

MP models determine the optimal solution by sequentially examining the entire solution space. Therefore, as a second advantage, **they consider all possible combinations of different types and quantities of transport assets, all cargoes, and all time periods.** Current simulation models consider cargoes one at a time with minimal or no "look ahead" to determine the ultimate effect of their immediate choices. Simulation models produce good solutions according to the quality of the embedded decision rules. However, it is unlikely that their solutions are even locally optimum let alone optimum over the entire range of possibilities.

Third, MP models produce a number of other measures about the system in addition to the optimal answer. For example, **they provide information on excess capacities in the system.** The additional capacities may include unused transportation assets and excess port capacities, or extra days between availability and actual loading or between desired and actual delivery. This information is useful when determining

[2] J. J. Jarvis and H. D. Ratliff, *Models and Concepts: System for Closure Optimization Planning and Evaluation (SCOPE)*, Document PDRC-83-06, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, 1983.

how robust the transportation system is in the face of numerous uncertainties. It can also be used to examine changes in various system parameters, such as the assignment of cargoes to ports or the determination of required available to load dates.

Fourth, other measures produced by MP models provide information about such fully utilized system assets as a aircraft type or a port's throughput capacity. For these assets, sensitivity measures (called *shadow prices*) **define the economic advantage (increase or decrease in the objective function) of having additional capacity.** For example, a shadow price will specify by how much the cost of transportation can be reduced if a cargo, or part of a cargo, is prepositioned or if the throughput capacity of a fully utilized port is increased, although they are good only for estimating the effect of small changes on the objective function. These sensitivity measures are useful for determining the economic effect of trade-offs in the system. For example, the measures can be used when evaluating whether to procure additional transportation assets or to allow cargoes to arrive at their destinations later than planned.

DIFFICULTIES THAT ARISE WITH MP FORMULATIONS[3]

Typical mathematical programming formulations involve optimizing a task easily. Their analytical solution, however, may or may not be attainable, depending on the size of the problem and the complexity of the solution algorithm. This difficulty confronted early efforts to apply mathematical programming techniques to strategic mobility problems.

To solve mathematical programming problems, computer algorithms will require a certain number of calculations. Some algorithms are *polynomially bounded,* meaning that the number of calculations required in the worst case that might arise is polynomial in the number of nodes, N. In simple terms, this means that a finite number of points are to be considered. But vehicle-routing problems, even without time windows (i.e., delivery restrictions), are what is known as *NP hard,* which means that no polynomially bounded algorithm has yet been found, and, worse, if a solution is suggested and claimed to be optimum, it would require as many calculations to verify optimality as it would to derive the optimal solution from scratch. Effectively, an infinite number of points must be considered. It has also been shown that simply obtaining a *feasible* solution to the transportation problem with time windows is *NP complete*: no polynomially bounded algorithm has yet been found, but the feasibility of a proposed solution can be verified in polynomially bounded time.

All of this simply means that NP hard algorithms tend to be exponential, requiring, in the worst case, a number of computations proportional to 2^N. Thus, every time a node is added, the problem size could double. The number of calculations for even "medium-sized" mobility problems (e.g., about 10,000 movements) could easily be 2×10^{90}, or many trillions of calculations. Problems of this size can overwhelm the

[3]See Appendix C of Schank, et al., 1991b, for a more complete discussion of the analytical complexity of applying mathematical programming to the strategic mobility problem and references to research that has been accomplished in related commercial areas, such as trucking and airline problems. Portions of this chapter were extracted from that appendix.

capabilities of even current computers and solution algorithms, resulting in solution times measured in days or even weeks.

Most of the decision support systems currently in use for related problems in non-military areas, such as vehicle routing and scheduling for trucking companies, shipping companies, and airlines, use heuristics, either by themselves or in conjunction with analytical solution techniques. While these heuristics usually result in acceptable operational policies—and, in general, these decision support systems are used for operational planning rather than for long-term resource planning—they do not necessarily result in optimal solutions. Even when the accuracy of the heuristics can be demonstrated, their use in resource planning is severely limited by their inability to perform the kinds of sensitivity analyses that exact, mathematical programming algorithms routinely provide.

The only meaningful rationale for using heuristics instead of analytically obtained, optimal solutions is the computational problem mentioned above: It simply may take too long to obtain the solution to the mathematical programming problem to meet the needs of the real-time operational environment. Even here, the mathematical programming solutions are needed to validate the proposed heuristic set of decision rules. To assess the closeness of the heuristic solutions to the optimal solutions, a realistic sample of operational situations would have to be presented to the heuristics and the generated solutions compared with those obtained from the programming model. Substantial deviations from optimality could demonstrate weaknesses in the heuristics and highlight where improvements are needed.

AGGREGATION TO REDUCE COMPUTATIONS

Rather than developing heuristic solutions, we have taken an alternative approach to solving the mathematical programming formulations. We have carefully examined the effect of aggregating different components of the data or of the system. Aggregating the number of cargoes, transportation assets, ports, or time periods reduces the size of the problem and, therefore, the solution time. Chapter Four describes our results of aggregating these different components.[4]

We believe this approach is sound because of the nature of the long-range requirements questions. Typically, these types of analyses look well into the future and address notional scenarios. Many of the specific data, such as the cargoes to be shipped or the times that cargoes are required in the theater, are estimates based on planning figures, or best guesses. Trying to capture more and more detail for these long-range problems overlooks the inherent uncertainty in many of the specific values.

Our research suggests that we can aggregate various components and still retain the essence of the problem. Our tests have shown that the answers we obtain from the aggregated models match exactly or very closely the results from the detailed model.

[4]Modeling experts exercise great caution in aggregating transportation assets, especially sealift, that have unique characteristics in terms of capacity, speed, or the type of cargo carried, e.g., ammunition ships.

And we can generate solutions to the aggregated problems in a matter of minutes, versus days or weeks for the detailed formulation.

We also have an advantage over the earlier efforts at applying mathematical programming to strategic mobility problems because of the impressive computational and algorithmic capabilities that have evolved in the past several years. Using current computer hardware and software, we can solve in reasonable times problems that could not be solved ten years ago.

SUMMARY

In summary, we believe that mathematical programming offers several advantages over traditional simulations in addressing transportation requirements questions, especially in examining the trade-offs that exist within the transportation system. The historical problem of computational complexity has been reduced somewhat by improved computer capabilities and solution algorithms. We reduce the complexity to a manageable state by aggregating certain data and system components. Our MP models provide reasonable answers to these requirements questions and can be used in conjunction with more-detailed simulation models to further understand the issues involved.

GENERAL MP FORMULATION

The product of our research is not a single mathematical formulation of the strategic mobility requirement question but rather a family of models, each member addressing one of the inherent trade-offs in the overall transportation system. It is easiest to begin describing the capabilities of these alternative models by starting with the basic cost-minimization model, which determines the least-cost set of transportation assets needed to deliver all cargoes on time.

This chapter provides an overview of the basic cost-minimization model. It first describes in general terms how we view the transportation system, including the assumptions embedded in the prototype. Then, it presents a small but detailed example of using the model. Finally, for those interested in the technical details, it gives the mathematical formulation of the model.

MODELING THE TRANSPORTATION SYSTEM

We view the transportation system as being composed of a number of objects, each with several attributes or characteristics. There are *cargoes*, or movement requirements, that must transit through the system. Each *movement* is composed of amounts of different types of cargo. Differentiating by type is important, because some transportation assets cannot carry all types of cargo. We define four types: bulk, oversized (will not fit on a C-130), outsized (will not fit on a C-141), and passengers (or "PAX"). We count the number of passengers and measure other cargo types by their weight and their volume, or "footprint."

Each cargo has two dates: The available to load date (ALD) indicates when the cargo is at a port and ready to be placed on a transportation asset; the required delivery date (RDD) specifies when the cargo must be at the theater port of entry. Therefore, there are also two locations specified for each cargo: the port of embarkation and the port of debarkation.

The mobility system is composed of a number of POEs and PODs. Each POE-POD pair represents a potential *channel* that the cargo moves over, although for most scenarios only a subset of the potential channels is used (that is, some POE-POD pairs are not linked as cargo channels). The *ports* have a geographical location plus a measure of throughput capacity, usually expressed in tons per day.

A number of different types of transportation assets move cargo. Each *asset* has a capacity for different types of cargo, a speed used to compute the time to travel from POE to POD, and a cycle time that defines when it can be reused. We use the capacities to determine load factors for the various types of assets. That is, we define for each cargo how many of each type of transportation asset are needed to move that cargo.

For transportation assets, we distinguish between those that are currently in the inventory and those that can be purchased or leased. For example, we may define the current transportation fleet as having a number of C-5, C-141, and KC-10 aircraft, and roll on/roll off (RoRo), bulk, and container ships. We may also define new aircraft (e.g., C-17s) or ships (e.g., a new, fast sealift ship) that can be bought and operated in addition to the existing fleet, and aircraft (e.g., CRAF) and ships (e.g., RRF) that can be rented for the duration of the scenario. Each of these additional assets has a cost[1] associated with its use.

The objective for the model is to minimize the total cost of acquiring (buying and operating or leasing) transportation assets over and above those that are part of the current inventory. The model has three sets of constraints—one representing the demand for transportation and two representing the supply of transportation assets.

The *demand constraints* specify that all cargoes must be delivered by their RDDs. For each cargo, we calculate a time window for each potential type of transportation asset (i.e., a cargo will have multiple time windows, one for each transportation asset that can carry that cargo). A *time window* defines the earliest and latest that the cargo can leave the POE (on that type of asset) and still arrive at its POD on time. The early end of the time window is the cargo's ALD; the late end is the RDD minus the load, transit, and unload times. We typically measure *time* in days, although *time steps* measured in hours or day multiples are also possible. The demand constraints then sum over all possible vehicles that can be used to transport the cargo and over all days in the time window. The total amount of the cargo shipped must be 100 percent.

The *first set of supply constraints* calculates the number of vehicles of a particular type on a particular channel on a particular day. We keep track of this number to ensure that there is an integer number of vehicles on a channel. The *second set of supply constraints* sums over all channels to find the total number of transportation assets used each day (or other appropriate time step).

A SIMPLE EXAMPLE

To show the numerical formulation and the results of the basic cost-minimization model, we use a very simple example containing only ten movement requirements

[1]Currently we use a 30-year life-cycle cost as the cost of buying new assets or of leasing commercial assets.

for a deployment from the United States to the Far East.[2] The various values for these movements appear in Table 3.1. This example involves only air transport, for which we consider C-141, C-5, and KC-10 aircraft. The capacities of these aircraft for various types of cargo and the relative cost factors are shown in Table 3.2. This example assumes that the relative cost factors are 1, 4, and 2 for the C-141, C-5, and KC-10, respectively. Sets, variables, and parameters appear in Table 3.3.

In this case, the objective function is to minimize cost. Given our assumption that the relative cost factors of C-141, C-5, and KC-10 aircraft are 1, 4, and 2, respectively, we multiply the number of new vehicles, Y_v, by their costs and sum them, giving

$$Y_{C141B} + 4Y_{C5} + 2Y_{KC10} \quad , \tag{3.1}$$

where

$$Y_i = \text{number of transportation assets of type } i\,.$$

The *load factors by cargo type*, calculated by dividing the weight of the cargoes (or number of passengers) by the capacity of the aircraft for that type of cargo, are shown

Table 3.1

Sample Movement Requirements

Movement, m	POE, $e(m)$	POD, $d(m)$	ALD, $A(m)$	RDD, $B(m)$	Bulk Tons	Oversized Tons	PAX
1	Seattle	Pingtung	C001	C002	15.0	0.0	0.0
2	Seattle	Chiayi	C001	C002	17.0	0.0	0.0
3	St. Louis	Pingtung	C001	C002	0.0	0.0	125.0
4	St. Louis	Taipei	C003	C005	0.0	43.0	75.0
5	St. Louis	Taipei	C004	C006	71.0	0.0	55.0
6	Boston	Tainan	C007	C010	21.0	0.0	27.0
7	New York	Tainan	C006	C009	37.5	0.0	25.0
8	San Francisco	Taipei	C007	C011	710.0	0.0	0.0
9	San Diego	Pingtung	C008	C012	377.0	0.0	0.0
10	San Francisco	Pingtung	C010	C013	0.0	22.0	0.0

Table 3.2

Sample Vehicle Capacities

Vehicle	Bulk Tons	Oversized Tons	Passengers	Cost Factors
C-141B	23.0	23.6	153	1
C-5	69.6	65.0	329	4
KC-10	62.1	26.4	257	2

[2]This example was taken from MAJ Stephen Cross, *A Proposed Initiative in Crisis Action Planning*, Defense Advanced Research Projects Agency, Arlington, Va., 1990. The actual computer code and output for this example are provided in Appendix A.

Table 3.3

Important Sets, Parameters, and Variables

Notation	Explanation
Sets	
v	Vehicle types
m	Movements
j	Cargo types
t	Time periods
Parameters	
$\$_v$	Per-unit cost of transportation assets of type v
B_{mv}	Latest ship date for cargo m on vehicle type v
A_m	Availability date of cargo m
L_{mjv}	Load factor[a] of cargo m of type j on vehicle v
S_{edv}	Cycle time for vehicle v to travel from POE e to POD d and return
U_{edtv}	The number of vehicles v loaded at POE e bound for POD d at time t
N_v	The number of vehicles v on hand
Variables	
Y_v	The number of transportation assets of type v
U_{edtv}	The number of vehicles v loaded at POE e bound for POD d at time t
X_{mjtv}	The number of vehicles v carrying cargo type j of movement m at time t

[a]The *load factor* is the weight of the cargoes (or number of passengers) divided by the capacity of the aircraft for that type of cargo.

in Table 3.4. For example, taking the 15 bulk tons from movement 1 in Table 3.1 and dividing it by the 23.0 bulk-ton capacity of the C-141B aircraft (Table 3.2) yields the 0.65 load factor that appears in the first row of Table 3.4.

Table 3.5 contains the delivery and cycle times. In this case, the delivery times are in hours rounded to the nearest day. Thus, for all aircraft, delivery time equals 1 day. *Cycle time* refers to the amount of time it takes for an aircraft to deliver a cargo and be ready to accept another load. In this case, all aircraft have a cycle time of 2 days, essentially the time required for a round trip.

We use the sample movement requirements in Table 3.1, the load factors in Table 3.4, and the cycle times in Table 3.5; set the number of available assets of a given type to be 1 ($N_v = 1 \forall v$); and set the relative costs factors of C-141Bs, C-5s, and KC-10s at 1, 4, and 2, respectively. The model consists of three sets of equations:

- Cost (the objective function)
- Demand for transportation
- Supply of transportation.

The objective function, Equation 3.1, is subject to the condition that the demand for transportation is satisfied. If we think in terms of the number of vehicles shipping a given cargo of a specific type on a particular day as being the *decision variable*, called

Table 3.4

Sample Load Factors, by Cargo Type

C-141 Loads for m	Bulk Tons	Oversized Tons	Passengers
1	0.65	0.00	0.00
2	0.74	0.00	0.00
3	0.00	0.00	0.82
4	0.00	1.82	0.49
5	3.09	0.00	0.36
6	0.91	0.00	0.18
7	1.63	0.00	0.16
8	30.87	0.00	0.00
9	16.39	0.00	0.00
10	0.00	0.93	0.00
C-5 Loads for m	Bulk Tons	Oversized Tons	Passengers
1	0.22	0.00	0.00
2	0.24	0.00	0.00
3	0.00	0.00	0.38
4	0.00	0.66	0.23
5	1.02	0.00	0.17
6	0.30	0.00	0.08
7	0.54	0.00	0.08
8	10.20	0.00	0.00
9	5.42	0.00	0.00
10	0.00	0.34	0.00
KC-10 Loads for m	Bulk Tons	Oversized Tons	Passengers
1	0.24	0.00	0.00
2	0.27	0.00	0.00
3	0.00	0.00	0.49
4	0.00	1.63	0.29
5	1.14	0.00	0.21
6	0.34	0.00	0.11
7	0.60	0.00	0.10
8	11.43	0.00	0.00
9	6.07	0.00	0.00
10	0.00	0.83	0.00

NOTE: Figures are rounded to the nearest 0.01.

Table 3.5

Sample Delivery and Cycle Times (days)

		Delivery Time			Cycle Time		
POE	POD	C-5	C-141	KC-10	C-5	C-141	KC-10
Boston	Tainan	1	1	1	2	2	2
New York	Tainan	1	1	1	2	2	2
St. Louis	Pingtung	1	1	1	2	2	2
St. Louis	Taipei	1	1	1	2	2	2
San Diego	Pingtung	1	1	1	2	2	2
San Francisco	Pingtung	1	1	1	2	2	2
San Francisco	Taipei	1	1	1	2	2	2
Seattle	Chiayi	1	1	1	2	2	2
Seattle	Pingtung	1	1	1	2	2	2

NOTE: Figures are rounded up to nearest day.

X_{mjtv} (where m is movement, j is type of cargo, t is time, and v is type of vehicle), then we can write the constraint that the first movement of bulk cargo be shipped in its entirety as

$$\frac{1}{0.65} X_{1,BULK,C001,C141B} + \frac{1}{0.22} X_{1,BULK,C001,C5} + \frac{1}{0.24} X_{1,BULK,C001,KC10} = 1 \quad . \tag{3.2}$$

That is, to convert the number of vehicles assigned to the fraction of the cargo shipped, each X_{mjtv} is multiplied by the reciprocal of the load factor and then summed to ensure that all the fractions of the cargo add up to 1.

The supply-of-transportation equation set comprises two sets of supply constraints. The first set sums the number of vehicles X_{mjtv} over the cargo types to show the number of vehicles used on a given channel on a given day:

$$\begin{aligned} X_{1,BULK,C001,C141B} &+ X_{1,OVER,C001,C141B} + X_{1,PAX,C001,C141B} \\ &- U_{SEATTLE,PINGTUNG,C001,C141B} \leq 0 \quad . \\ &\cdot \\ &\cdot \\ &\cdot \end{aligned} \tag{3.3}$$

The second set sums across all channels for every day,

$$\begin{aligned} U_{SEATTLE,CHIAYI,C001,C141B} &+ U_{ST-LOUIS,CHIAYI,C001,C141B} \leq 1 + Y_{C141B} \quad , \\ &\cdot \\ &\cdot \\ &\cdot \end{aligned} \tag{3.4}$$[3]

yielding the minimum-cost set of transportation assets. The solution also yields a complete schedule for shipping the movements.

This sample problem can be solved in two ways, either by linear programming, which allows the number of each type of vehicle to be expressed as fractions, or by integer programming, which additionally restricts X_{mjtv} and U_{edtv} to be whole numbers to ensure that Y_v is expressed as whole numbers of aircraft. Table 3.6 shows the sample integer and linear solutions and their associated minimum cost factor. Note that if the linear solution were to be rounded up it would be identical to the integer solution.

Tables 3.7 through 3.10 show the integer and linear schedules. The vehicle schedules show the number of vehicles of a given type on a particular channel on a particular day; the cargo schedules show how each particular cargo is shipped. Note that while both the linear and integer approaches produce similar answers for the total number of assets used, both the vehicle and cargo schedules look quite different.

[3] Not every calculation is included. Dots indicate that a series of calculations like those shown have been carried out.

Table 3.6

New Assets Required

Vehicle	Integer Solution	Linear Solution
C-141B	0	0
C-5	0	0
KC-10	4	3.71
Total Cost	8[a]	7.42

[a]One KC-10 has a cost factor of 2.

Table 3.7

Integer Vehicle Schedule

POE	POD	Day	Vehicle Type	Number of Vehicles	Number Available	New Assets Required
SEATTLE	PINGTUNG	C001	KC10	1	1	0
SEATTLE	CHIAYI	C001	KC10	1	1	0
ST-LOUIS	PINGTUNG	C001	C5	1	1	0
ST-LOUIS	TAIPEI	C003	C5	1	1	0
ST-LOUIS	TAIPEI	C004	KC10	1	1	0
ST-LOUIS	TAIPEI	C005	C5	1	1	0
BOSTON	TAINAN	C007	C5	1	1	0
NEW-YORK	TAINAN	C006	C141B	1	1	0
NEW-YORK	TAINAN	C008	C141B	1	1	0
SAN-FRAN	PINGTUNG	C012	C141B	1	1	0
SAN-FRAN	TAIPEI	C007	KC10	5	1	4
SAN-FRAN	TAIPEI	C009	C5	1	1	0
SAN-FRAN	TAIPEI	C009	KC10	5	1	4
SAN-FRAN	TAIPEI	C010	C141B	1	1	0
SAN-DIEG	PINGTUNG	C011	C5	1	1	0
SAN-DIEG	PINGTUNG	C011	KC10	5	1	4

Tables 3.9 and 3.10 present the cargo schedules. The tables are easily interpreted. For example, in the integer solution, the bulk portion of movement 5 is loaded into 0.786 of a KC-10 (48.8 tons) on day C004 and 0.319 of a C-5 (22.2 tons) on day C005; the entire oversized portion of movement 5 is shipped on a C-5 on day C005; and the entire allotment of passengers for movement 5 is shipped on a KC-10 on day C004.

In this case, the integer solution takes only a few more seconds to calculate than the linear solution, but in some larger problems the integer solution can take days to calculate; a very close linear solution (i.e., one producing results close to those of the integer solution) can be solved in less than one hour. The linear solution has additional advantages, a primary one being that as a by-product it gives numbers that can help in sensitivity analysis.

Table 3.11 shows the sensitivity analysis for the above problem. The "Shadow Price" column shows the decrease in the cost at the margin if the associated cargo is not shipped. A shadow price of zero indicates that the system is not used to capacity and, thus, prepositioning cargo, for example, would not have any effect on the least-cost solution. Values other than zero suggest an effect on the solution from preposi-

Table 3.8

Linear Vehicle Schedule

POE	POD	Day	Vehicle Type	Number of Vehicles
SEATTLE	PINGTUNG	C001	C5	0.216
SEATTLE	CHIAYI	C001	C5	0.244
ST-LOUIS	PINGTUNG	C001	C5	0.380
ST-LOUIS	TAIPEI	C003	C5	0.679
ST-LOUIS	TAIPEI	C003	KC10	0.292
ST-LOUIS	TAIPEI	C004	C5	0.321
ST-LOUIS	TAIPEI	C004	KC10	0.237
ST-LOUIS	TAIPEI	C005	C5	0.679
BOSTON	TAINAN	C007	C141B	0.176
BOSTON	TAINAN	C007	C5	0.243
BOSTON	TAINAN	C009	C141B	0.176
NEW-YORK	TAINAN	C006	C141B	0.824
NEW-YORK	TAINAN	C006	C5	0.321
SAN-FRAN	PINGTUNG	C012	C141B	0.049
SAN-FRAN	PINGTUNG	C012	C5	0.321
SAN-FRAN	TAIPEI	C007	C5	0.436
SAN-FRAN	TAIPEI	C007	KC10	4.700
SAN-FRAN	TAIPEI	C008	C141B	0.323
SAN-FRAN	TAIPEI	C009	C5	0.679
SAN-FRAN	TAIPEI	C009	KC10	4.700
SAN-FRAN	TAIPEI	C010	C141B	0.824
SAN-FRAN	TAIPEI	C010	C5	0.321
SAN-DIEG	PINGTUNG	C008	C141B	0.500
SAN-DIEG	PINGTUNG	C008	C5	0.321
SAN-DIEG	PINGTUNG	C011	C141B	0.176
SAN-DIEG	PINGTUNG	C011	C5	0.679
SAN-DIEG	PINGTUNG	C011	KC10	4.700

tioning. For example, movement 8 has a shadow price of –7.663, which indicates that prepositioning some or all of this cargo would have a very large effect on the least-cost solution; movement 1 has a price of zero, which shows that even if all of movement 1 were prepositioned, it would not change the solution. The "Allowable Increase" and "Allowable Decrease" columns show by how much a cargo can be increased or decreased without changing the solution at all. For example, the number of passengers in movement 5 can be increased by 2080 percent without changing the least-cost solution.

MATHEMATICAL FORMULATION OF THE COST-MINIMIZATION MODEL

The objective function for our cost-minimization model is

$$\sum_{v} \$_v Y_v \ . \tag{3.5}$$

Table 3.9

Integer Cargo Schedule

Movement	Cargo Type	Day	Vehicle Type	Vehicle Loads
1	BULK	C001	KC10	0.242
1	PAX	C001	KC10	0.758
2	BULK	C001	KC10	0.274
2	PAX	C001	KC10	0.726
3	OVER	C001	C5	0.62
3	PAX	C001	C5	0.38
4	BULK	C003	C5	0.11
4	OVER	C003	C5	0.662
4	PAX	C003	C5	0.228
5	BULK	C004	KC10	0.786
5	BULK	C005	C5	0.319
5	OVER	C005	C5	0.681
5	PAX	C004	KC10	0.214
6	BULK	C007	C5	0.302
6	OVER	C007	C5	0.616
6	PAX	C007	C5	0.082
7	BULK	C006	C141B	1
7	BULK	C008	C141B	0.63
7	OVER	C008	C141B	0.206
7	PAX	C008	C141B	0.163
8	BULK	C007	KC10	5
8	BULK	C009	C5	0.948
8	BULK	C009	KC10	5
8	BULK	C010	C141B	1
8	PAX	C009	C5	0.052
9	BULK	C011	C5	0.955
9	BULK	C011	KC10	5
9	PAX	C011	C5	0.045
10	OVER	C012	C141B	0.932
10	PAX	C012	C141B	0.068

The first set of constraints, ensuring that 100 percent of the cargo requirements is shipped on time, is

$$\sum_{v \supset L_{mjv}>0} \sum_{t=A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} = 1 \tag{3.6}$$

for all m,j such that $L_{mjv} > 0$ for at least one v.

The second set of constraints, ensuring that an integer number of vehicles is used on each channel on each day, is

$$\sum_{m \in \{e,d\}} \sum_{t=A_m}^{B_{mv}} X_{mjtv} - U_{edtv} \leq 0 \tag{3.7}$$

for all $e \in E, d \in D, t \in T, v \in V$ such that channel $\{e, d\}$ is in use at time t.

Table 3.10

Linear Cargo Schedule

Movement	Cargo Type	Day	Vehicle Type	Vehicle Loads
1	BULK	C001	C5	0.216
2	BULK	C001	C5	0.244
3	PAX	C001	C5	0.380
4	OVER	C003	C5	0.662
4	PAX	C003	KC10	0.292
5	BULK	C004	C5	0.321
5	BULK	C004	KC10	0.023
5	BULK	C005	C5	0.679
5	PAX	C004	KC10	0.214
6	BULK	C007	C5	0.243
6	BULK	C009	C141B	0.176
6	PAX	C007	C141B	0.176
7	BULK	C006	C141B	0.660
7	BULK	C006	C5	0.321
7	PAX	C006	C141B	0.163
8	BULK	C007	C5	0.436
8	BULK	C007	KC10	4.700
8	BULK	C008	C141B	0.323
8	BULK	C009	C5	0.679
8	BULK	C009	KC10	4.700
8	BULK	C010	C141B	0.824
8	BULK	C010	C5	0.321
9	BULK	C008	C141B	0.500
9	BULK	C008	C5	0.321
9	BULK	C011	C141B	0.176
9	BULK	C011	C5	0.679
9	BULK	C011	KC10	4.700
10	OVER	C012	C141B	0.049
10	OVER	C012	C5	0.321

Table 3.11

Sample Sensitivity Analysis

Movement, *m*	Cargo Type	Shadow Price	Allowable Decrease	Allowable Increase
1	Bulk	0.000	1.00	0.73
2	Bulk	0.000	1.00	0.67
3	Passengers	0.000	1.00	0.42
4	Oversize	0.000	0.07	0.21
	Passengers	0.000	1.00	15.06
5	Bulk	0.000	0.04	0.13
	Passengers	0.000	0.25	20.80
6	Bulk	–0.225	1.00	0.53
	Passengers	–0.046	0.11	4.56
7	Bulk	–0.399	0.09	0.26
	Passengers	–0.039	1.00	0.12
8	Bulk	–7.663	0.55	0.14
9	Bulk	–4.040	0.13	2.07
10	Passengers	0.000	1.00	0.76

NOTE: Figures are rounded to the nearest 0.01.

The third set of constraints, calculating the total number of transportation assets used each day, is

$$\sum_{e}\sum_{d}\sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \leq N_v \tag{3.8}$$

for all $h \in T, v \in V$.

The next chapter addresses data and aggregation issues.

Chapter Four

DATA AND AGGREGATION ISSUES

We indicated in Chapter Two that one of the drawbacks to mathematical programming solutions is that the number of calculations required increases exponentially as a function of the number of variables. Although advances in computers and solution algorithms have partially offset this disadvantage, large transportation problems can still pose computational challenges, requiring a long time to solve. This chapter describes how we applied aggregation techniques to reduce the number of calculations required while retaining the accuracy and other advantages of mathematical programming. It illustrates the process by using data from an ARPA data set.

THE DIFFICULTY OF SOLVING LARGE MATHEMATICAL PROGRAMMING PROBLEMS

It is difficult to know exactly how long it can take to solve a given model, because solution time depends on the complexity of the model and unique characteristics of the data. However, we do know that solution time increases more than linearly with the number of nonzero variables. We can estimate an upper bound for such variables and, thus, the difficulty of a problem, by counting the number of all variables implied by the system of equations used in the model. The formulas given below were derived from the specific equations for the cost-minimization model; however, the implications for aggregation also hold for the rest of the family of strategic mobility models discussed in this report.

For purposes of illustration, we use statistics from a medium-sized data set provided by the Advanced Research Projects Agency to participants in the ARPA–Rome Lab Transportation Planning and Scheduling Initiative. The data pertain to a low-intensity conflict outside the continental United States (CONUS).

If we used the raw data "as is," the problem would be virtually impossible to solve using mathematical programming techniques and current computer technology. The number of variables in the ARPA data set could exceed 10 million.[1] (The formula appears at the end of this chapter). Solution of a problem with this many variables would consume considerable computation time. Therefore, we sought a way to

[1]This number represents the most pessimistic upper bound. The real upper bound will depend on the compositon of the movement requirement. For example, for the data sets we worked with, the number of variables was typically 20 percent of the upper bound.

collapse the amount of detail in the formulation without affecting the accuracy of the final solution. To that end, we examined a number of approaches to aggregation. They included

- combining cargoes with similar characteristics into packages of movement requirements. A *package* merges into one movement requirement all cargoes that arrive at a given POE at the same time and are required to be delivered at the same POD by the same time.
- aggregating the number of channels by considering ports in complexes (POEs or PODs in a geographic region) instead of treating them as individual ports
- reducing the number of types of transportation assets considered
- reducing the number of time periods by taking large time steps or by concentrating only on the peak periods.

However, we can rewrite the problem to make solving it easier. In our analysis, we followed the general principle that any modification of the problem that does not change the *binding constraints* (that is, any of the factors that drive the solution, such as moving the largest cargo under the most demanding time requirements) will not change the answer. For each method, we ran the disaggregated model to obtain a solution and then reran the model with aggregated categories, comparing the solutions from each. Although this methodology was practical only for small scenarios (several dozen movement requirements), it enabled us to discover what types of aggregation appear to have little or no effect on the final solution. In addition, because of the mathematical formalism of the model, we can *prove* that certain types of aggregation have no effect on the final solution, and we can place upper bounds on the loss of precision to be expected from other types of aggregation.

We discovered that the following modifications did not affect the solution:

- Aggregating movement requirements with identical POEs, PODs, ALDs, LADs, and cargo types
- Considering only the peak-period requirements.

However, some modifications may change the solution:

- Aggregating channels (POE-POD pairs)
- Choosing larger basic time units
- Aggregating similar vehicle types.

In the ARPA scenario, aggregating similar movement requirements, using only the peak period, and using only ten channels reduce the problem size from over 10 million variables to fewer than 30,000 variables (a reduction of more than 97 percent). The solution to the aggregated problem is identical to that of the original problem. While the original problem required an extremely long time to solve, the smaller problem can be solved in a few minutes on a typical workstation.

TYPES OF AGGREGATION

The following subsections describe the aggregation process in general terms. A more technical explanation, including the formulas, appears at the end of this chapter.

Movements

In our model, grouping those movements with identical POEs, PODs, ALDs, LADs, and cargo types will not change the final answer, because the mathematical program implicitly sums together all the constraints that correspond to a particular combination of POE, POD, ALD, LAD, and cargo type when determining the demand for transportation assets on a particular channel in a particular time period. If this type of aggregation is done, the final solution can be disaggregated to determine the effect of a particular cargo of interest, without any resultant loss of information.

Ports

Aggregating ports reduces the total number of channels. Such aggregation can affect the fidelity of the solution in two ways. First, aggregating *bottleneck ports* (ports where desired capacity exceeds actual capacity for one or more days) may result in a larger *virtual port,* a port that does not accurately reflect the capacity constraints of the component ports. Second, if two aggregated ports are active during the same time period, aggregating ports may result in undercounting the number of transportation requirements. Undercounting occurs because "fractional" vehicles may be added together and rounded up to give erroneous answers (e.g., assigning half a C-5 to Seattle-Tokyo and half a C-5 to Seattle-London on the same day, implies a need for two C-5s, not one).

However, aggregating ports that are close geographically may result in a more realistic model, because it is a simple way of treating multiporting, i.e., when a vehicle may stop at two closely located POEs to pick up cargo before proceeding to distant PODs. This method of approximation should be treated with great caution to avoid the bottleneck and undercounting pitfalls mentioned above.

Even though the problems mentioned above exist, we *may* safely group together channels that are never active during the same time period, as long as the channels have identical travel-time characteristics and identical capacities. Such grouping is slightly counterintuitive, because it means that two channels serving vastly different geographic areas can be lumped together within the model if they do not contend for the same transportation assets. Examining the model equations, however, reveals that we need unique identifiers only for the channels active during a given time period; swapping channel labels does not affect the mathematics of the model at all and can greatly reduce problem size. For example, in the problem illustrated in the preceding chapter, we may aggregate the Seattle-Chiayi and the Boston-Tianan channels, because the Seattle-Chiayi channel is active only during days C001 and C002, and the Boston-Tianan channel is active on days C007 through C010. Since these two channels never contend for the same transportation assets, combining them produces a problem that gives an identical solution to the original problem.

Time

Aggregation in time reduces the number of periods and thus the total number of variables. Choosing large basic time periods will cause the program to overestimate the number of vehicles required, but the overestimation will not necessarily be great. It will be no greater than the ratio between cycle time rounded to the next higher unit and cycle time, as long as the cargo availability and due dates can be measured exactly in the larger units. Thus, if we knew availability and due dates to the day, and if it took a transportation asset 1.8 days to cycle, the overestimate would not be any larger than 2.0:1.8 (or 0.1).

Vehicles

Aggregating vehicles also reduces the number of variables. Aggregating two different types of vehicles with differing capacities (but that are otherwise identical) by describing them as one type with the minimum capacity of the two vehicles will cause the program to overestimate the number of vehicles needed by *at worst* the ratio of the two capacities. (Thus, using the bulk-ton carrying capacity of C-5s and KC-10s shown in Table 3.2, the program would overestimate the requirement by no more than a factor of 0.1 (69.6/62.1 = 1.1). This small factor is due to the direct relationship between the carrying capacities of vehicles and their corresponding load factors for a given movement; the load factors, in turn, determine the number of transportation assets required.

Peak Period Versus Whole Scenario

In our model it is sufficient to consider only the peak period when solving for the number of transportation assets required. The *peak period* is defined as that period of time that includes all the binding cargo constraints.[2] Since the peak period, by definition, includes the binding constraints, this reduced model is equivalent to the full model. To avoid any "tail effects," we include the cycle time around the peak.

The question is how to find the peak period without solving the disaggregate model in the first place. In practice, this is done by looking at the desired capacity (in tons per day and PAX per day) for each channel for each day, summing across all channels, and finding the time interval that has the heaviest traffic. Once the model has been solved for the candidate peak period, work can be double-checked by slightly expanding the period covered to see if the answer changes. If not, the whole peak has probably been captured. In the example in Chapter Three, the peak period occurs between days C008 and C012.

[2]For this example, peak airlift demand coincided with that of sealift, but this will not necessarily always be the case. When the periods do not coincide, we include the cargo requirements for both peaks and treat them as a single peak period.

TECHNICAL DESCRIPTION OF THE AGGREGATION PROCESS

The number of variables for the simple cost-minimization model is given by the formula

$$\begin{aligned}&\text{Vehicle\_Types} \times \text{Periods} \times \text{Channels} + \\ &\text{Vehicle\_Types} \times \text{Periods} \times \text{Movements} \times \text{Cargo\_Types} + \\ &\text{Vehicle\_Types} + 1. \end{aligned} \tag{4.1}$$

Inserting the specific numbers from the ARPA data set, we get

$$\begin{aligned} & 8 \times 36 \times 47 + \\ & 8 \times 36 \times 9102 \times 4 + \\ & 8 + 1 \\ = \quad & 10{,}499{,}049 \text{ variables.} \end{aligned}$$

Movements

The process of aggregating movements reduces the problem size proportionally to

$$\text{Vehicle\_Types} \times \text{Periods} \times \text{Cargo\_Types} \tag{4.2}$$

(which is simply the partial derivative of the equation for the number of variables with respect to the number of movements). In the ARPA example, grouping cargoes in this fashion reduces the number of movements from 9102 to 400, and as a result reduces the number of variables by 10,024,704.

Placing these variables in mathematical terms, we can see that this reformulation is equivalent by examining the constraint that states that 100 percent of the cargo requirements will be shipped on time. This constraint is of the form

$$\sum_{v \supset L_{mjv} > 0} \sum_{t = A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} = 1 \quad , \tag{4.3}$$

where L_{mjv} is the load factor of cargo m of type j on vehicle v, and X_{mjtv} measures the number of cargo-loads of cargo m of type j on vehicle v at time t. Now, let L_1 and L_2 be the load factors for two different cargoes that share the same POEs, PODs, ALDs, LADs, and cargo types. Clearly, the following constraints,

$$\sum_v \sum_t \frac{1}{L_1} X_1 = 1 \text{ and } \sum_v \sum_t \frac{1}{L_2} X_2 = 1 \quad ,$$

are equivalent to the following constraint:

$$\sum_{v}\sum_{t}\frac{1}{(L_1 + L_2)}(X_1 + X_2) = 1 \tag{4.4}$$

for the solution variables of interest.

The type of aggregation described for movements (and for ports) is a standard practice in simulation models. We have simply applied that practice to our prototype.

Channels

Reducing the number of channels reduces the size of the problem proportionally to

$$\text{Vehicle\_Types} \times \text{Periods} \tag{4.5}$$

(the partial derivative with respect to Channels). In our example, reducing the number of channels from 47 to 10 reduces the number of variables by 10,656 but does not change the answer because the binding constraints remain the same, even given the high degree of port aggregation.

Time

This principle of aggregation can be derived by observing that the demand for a particular vehicle caused by a particular cargo equals the size of the cargo divided by the number of periods between the first and last period it can be shipped; e.g., we can choose X_{mjtv} such that the cargo constraint of Equation 4.3,

$$\sum_{v \supset L_{mjv} > 0}\ \sum_{t = A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} = 1 \quad ,$$

is consistent with

$$\frac{L_{mjv}}{(B_{mv} - A_m)} = X_{mjtv} \text{ for particular } m, t, v, \text{ and } j. \tag{4.6}$$

The supply of a particular vehicle for this cargo is, roughly speaking, the demand multiplied by the cycle time (i.e., one C-5 load of cargo translates into two C-5-days of transport assets required). Thus, the precision of the cycle time determines the amount by which the model will over- or underestimate the transportation requirements, as can be seen from the fact that the two constraints that regulate transportation supply,

$$\sum_{m \in \{e,d\}}\ \sum_{t = A_m}^{B_{mv}} X_{mjtv} - U_{edtv} \le 0 \tag{4.7a}$$

and

$$\sum_{e}\sum_{d}\sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \le N_v \ , \tag{4.7b}$$

imply that

$$\sum_{e}\sum_{d}\sum_{m\in\{e,d\}}\sum_{t=h-S_{edv}+1}^{h} X_{mjtv} \le N_v + Y_v \quad . \tag{4.8}$$

If we fix m, then this relationship can be reduced to

$$\sum_{t=h-S_{edv}+1}^{h} X_{mjtv} \le N_v + Y_v \ . \tag{4.9}$$

The notation on the summation sign shows that if a transportation asset is assigned at a time t, it does not become available again until time $(t + S_{edv})$. The right-hand side of the equation is the total demand for transportation, so the relationship between the cycle time S_{edv} and the number of assets needed is clear. In the limiting case, assuming that X_{mjtv} is constant implies that $X_{mjtv}\ S_{edv} \le N_v + Y_v$; i.e., the number of transportation assets is directly dependent on the precision of the cycle-time measurement.

This technique reduces the size of the problem proportionally to

$$\begin{aligned}&\text{Vehicle\_Types} \times \text{Channels} + \\ &\text{Vehicle\_Types} \times \text{Movements} \times \text{Cargo\_Types}\ .\end{aligned} \tag{4.10}$$

In our example, changing the basic time unit from 1 day to 2 days reduces the size by 5,249,524 variables and, on the test data set, produces results identical to those of the original problem. However, since the availabilities and due dates for the cargoes do not conveniently fall either on all-odd or all-even days, this reformulation is not, strictly speaking, equivalent to the original problem. However, the binding constraints (i.e., those cargoes that drive the answer) were not affected by this change in precision, so the aggregate model produces identical answers.

Vehicles

In our model, this step reduces the problem proportionally to

$$\begin{aligned}&\text{Periods} \times \text{Channels} + \\ &\text{Periods} \times \text{Movements} \times \text{Cargo\_Types} + 1\end{aligned} \tag{4.11}$$

(the partial derivative with respect to Vehicle_Types). In our example, reducing the number of vehicle types from 8 to 4 reduces the size of the problem by 5,249,524 variables.

Peak Period Versus Whole Period

Any contiguous subset of the days of the scenario can be selected as long as it includes the interval

$$\begin{aligned}&[\text{earliest peak day} - \text{longest cycle time},\\ &\text{latest peak day} + \text{longest cycle time}].\end{aligned}$$

Cutting the number of periods considered reduces the size of the problem proportionally to

$$\begin{aligned}&\text{Vehicle\_Types} \times \text{Channels} +\\ &\text{Vehicle\_Types} \times \text{Movements} \times \text{Cargo\_Types}\end{aligned} \tag{4.12}$$

(the partial derivative with respect to Periods). In the example, considering a subset of only 10 days reduces the problem size by about 75 percent.

Combined Effects of Aggregation and Subsetting on Problem Size

Combining several different types of aggregation has dramatic effects on the problem size. Considering a subset of only 10 days and only 10 channels allows us to group the original 9102 movements into just 83 movements, reducing the problem size from 10,499,049 variables to

$$\begin{aligned}&8 \times 10 \times 10 +\\ &8 \times 10 \times 83 \times 4 +\\ &8 + 1\\ =\ &27{,}369 \text{ variables.}\end{aligned}$$

Note that this reformulation is mathematically equivalent to the disaggregate formulation, and thus the answer to the smaller problem is *identical* to the answer to the larger problem. Aggregation need not imply a loss of precision, if done properly.

This process reduces the problem size by a factor of

$$\text{Vehicle\_Types} \times \text{Periods} \times \text{Cargo\_Types} \tag{4.14}$$

(which is simply the partial derivative of the equation for the number of variables with respect to the number of movements). In the ARPA example, grouping cargoes in this fashion reduces the number of movements from 9102 to 400, and as a result reduces the number of variables by 10,024,704.

Chapter Five

TRADE-OFFS BETWEEN EARLINESS, LATENESS, PREPOSITIONING, AND COST

Cost-minimization models have limited utility. In many cases, even the minimum-cost solution may yield a dollar figure that is wildly out of the question. A more realistic approach is to have a cost constraint in the model, set either to a reasonable figure or range of figures, and to examine trade-offs between expenditures on transportation and cargo lateness, improved availability, and prepositioning. One of the attractive characteristics of the model described in Chapter Three is its ability to examine exactly these trade-offs. This chapter describes how the model can be modified to investigate different approaches to the transportation requirement and provides the necessary mathematical formulations.

The model is modified by taking the basic framework provided by the minimize-cost model, adding a cost constraint, changing the objective function to track the aspect of interest, and adding supplementary variables to the constraints. This process involves changing just a few equations; the data structures and requirements remain the same.

For example, to change the minimize-cost model to one that minimizes some measure of lateness, we take the cost, which used to be the objective function, and constrain it to be less than some cost figure. Then, to allow cargo to be shipped late, we add a variable to the equation, a variable that normally requires that 100 percent of the cargo be shipped on time. The old variable that kept track of when cargo was shipped on time is retained, and the new variable keeps track of late cargo. Then we use this new variable in the new objective function, multiplying it by the number of days late so that we can keep track of the total number of ton-days late and thus minimize this common measure of lateness.[1]

Changing the model to minimize earliness is done in exactly the same way, except in this case the new auxiliary variable is constrained to carry only early cargo. It is the mirror image of the minimize-lateness model.

Changing the model to minimize "prepo" is even simpler. In this case, the auxiliary variable does not need to track the date when cargo that cannot make it on time is

[1] *A ton-day* is a measure of timeliness (early or late) and is simply the number of tons of cargo multiplied by the number of days.

shipped, and the objective function just adds up all the tonnage that needs to be prepositioned.

Additional models can be made that simultaneously optimize over prepositioning, earliness, lateness, and cost. However, this process requires specifying an objective function that evaluates each alternative correctly. That is, doing the job properly requires some idea of the penalty or advantage of a cargo being available a day early or a day late, so that the objective function can make intelligent choices between alternatives.

MATHEMATICAL FORMULATIONS

Below are listed some alternative model formulations based on the original minimize-cost model, which we compare directly with the original equations.

The Minimize-Lateness Model

This model is designed to look at the possibility of relaxing the constraints on on-time delivery of cargoes and to find the shipping schedule that minimizes the number of ton-days late. We do this by widening the time window within which a cargo can be shipped, allowing cargoes to be delivered after the desired delivery dates, and keeping track of whether cargo is shipped on time so that we can put the late cargo in a penalty function that we can try to minimize. Cargo that is shipped on time is represented by the X variable, as it has been in all the examples above, and cargo that is shipped late is represented by a very similar variable, W.

The objective function of the minimize-lateness model keeps track of the number of ton-days late:

$$L = \sum_{m} \sum_{j} \sum_{t>B_{mv}} \sum_{v} (t - B_{mv}) K_{vj} W_{mjtv} \quad . \tag{5.1}$$

The tonnage figure is the $K_{vj}W_{mjtv}$ term in Equation 5.1, where W_{mjtv} is the variable that keeps track of late shipments and K_{vj} is a constant to convert the figure from vehicle-loads into tons. To get the ton-days figure, the tonnage figure is multiplied by the term $(t - B_{mv})$, where t is the time when the cargo actually gets shipped and B_{mv} is the latest date a cargo can be shipped and still arrive on time.

Equation 5.2 is the cost constraint, which bears more than a passing resemblance to the objective function of the minimize-cost model. Here, B is the cost limit:

$$\sum_{v} \$_v Y_v \leq B \quad . \tag{5.2}$$

Next is the constraint that 100 percent of the cargo be shipped. The X_{mjtv} term tracks the cargo shipped on time; the W_{mjtv} term tracks the late cargo:

$$\sum_{v \supset L_{mjv}>0} \sum_{t=A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} + \sum_{v \supset L_{mjv}>0} \sum_{t \geq B_{mv}+1} \frac{1}{L_{mjv}} W_{mjtv} = 1 \quad . \tag{5.3}$$

The channels constraint has an extra term to account for the late cargo:

$$\sum_{m \in \{e,d\}} \sum_{t} (X_{mjtv} + W_{mjtv}) - U_{edtv} \leq 0 \; . \tag{5.4}$$

This equation is identical to the one in the cost-minimization model, Equation 4.7b:

$$\sum_{e} \sum_{d} \sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \leq N_v \quad . \tag{5.5}$$

The Minimize-Earliness Model

This model is the mirror image of the minimize-lateness model above, and it works in a similar fashion. Instead of relaxing delivery dates, here we relax *availability* dates. We do so by widening the time window within which a cargo can be shipped, allowing cargoes to be available *before* the desired availability dates, and keeping track of whether cargo is shipped on time or not so that we can put the late cargo in a penalty function that we can try to minimize. Cargo that is shipped on time is represented by the X variable, and cargo that is shipped early is represented by a very similar variable, W.

The objective function of the minimize-earliness model keeps track of the number of ton-days early. The tonnage figure is the $K_{vj}W_{mjtv}$ term in Equation 5.6, where W_{mjtv} is the variable that keeps track of early shipments and K_{vj} is a constant to convert the figure from vehicle-loads into tons. To get the ton-days figure, the tonnage figure is multiplied by the term $(A_m - t)$, where t is the time the cargo actually gets shipped and A_m is the earliest date a cargo is said to be available:

$$Z = \sum_{m} \sum_{j} \sum_{t \geq A_m - \tau}^{A_m - 1} \sum_{v} (A_m - t) K_{vj} W_{mjtv} \quad , \tag{5.6}$$

where τ is the maximum number of time periods we will allow the model to ship early. Again, B is the cost limit:

$$\sum_{v} \$_v Y_v \leq B \quad . \tag{5.7}$$

Equation 5.8 is the constraint that 100 percent of the cargo be shipped. The X_{mjtv} term tracks the cargo shipped on time; the W_{mjtv} term tracks the early cargo:

$$\sum_{v \supset L_{mjv}>0} \sum_{t=A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} + \sum_{v \supset L_{mjv}>0} \sum_{\substack{t<A_m \\ t<B_{mv} \\ t \geq A_m-\tau}} \frac{1}{L_{mjv}} W_{mjtv} = 1 \quad . \tag{5.8}$$

The rest of the constraints are almost identical to their counterparts above, except that they pertain to early cargo rather than to late cargo:

$$\sum_{v \supset L_{mjv}>0} \sum_{t=A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} + \sum_{v \supset L_{mjv}>0} \sum_{t<A_m} \frac{1}{L_{mjv}} W_{mjtv} = 1 \quad . \tag{5.9}$$

$$\sum_{m \in \{e,d\}} \sum_{t} (X_{mjtv} + W_{mjtv}) - U_{edtv} \leq 0 \quad . \tag{5.10}$$

$$\sum_{e} \sum_{d} \sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \leq N_v \quad . \tag{5.11}$$

The Minimize-Prepositioning Model

The minimize-prepositioning model is very similar to the minimize-lateness and minimize-earliness models, except that here we assume that when a cargo cannot be shipped on time, it can always be prepositioned. This model parallels those above, except, in this case, *when* the W cargoes get shipped is unimportant (and thus we can drop the t subscript on W), and we assume that the prepositioned cargoes do not contend for transportation assets with the other cargoes.

The objective function of the minimize-prepo model, Equation 5.12, keeps track of the number of tons that must be prepositioned. The tonnage figure is $K_{vj}W_{mjv}$, where W_{mjv} is the variable that keeps track of prepositioned shipments, and K_{vj} is a constant to convert the figure from vehicle-loads into tons. The rest of the model is similar to the models above:

$$Z = \sum_{m} \sum_{j} \sum_{v} K_{vj} W_{mjv} \quad . \tag{5.12}$$

$$\sum_{v} \$_v Y_v \leq B \quad . \tag{5.13}$$

$$\sum_{v \supset L_{mjv}>0} \sum_{t=A_m}^{B_{mv}} \frac{1}{L_{mjv}} X_{mjtv} + \sum_{v \supset L_{mjv>0}} \frac{1}{L_{mjv}} W_{mjv} = 1 \quad . \tag{5.14}$$

$$\sum_{m \in \{e,d\}} \sum_{t=A_m}^{B_{mv}} X_{mjtv} - U_{edtv} \leq 0 \quad . \tag{5.15}$$

$$\sum_{e} \sum_{d} \sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \leq N_v \quad . \tag{5.16}$$

$$\sum_{m \in \{e,d\}} \sum_{t=A_m}^{B_{mv}} X_{mjtv} - U_{edtv} \leq 0 \quad . \tag{5.15}$$

$$\sum_{e} \sum_{d} \sum_{t=h-S_{edv}+1}^{h} U_{edtv} - Y_v \leq N_v \quad . \tag{5.16}$$

EXTENDED EXAMPLE OF THE MP MODEL

To demonstrate the capabilities of the MP model, we used a data set provided by the JS/J-4. This scenario is an early version of one analyzed during the Mobility Requirements Study, a congressionally directed study performed by the Joint Staff to define transportation requirements for the future.

Table 6.1 shows the various characteristics of the scenario and corresponding values for our demonstration runs. We aggregated the almost 5800 cargoes of the scenario into slightly fewer than 500. Most of the decrease results from combining cargoes with the same ALD, RDD, POE, and POD into a package of cargoes.

We also examined only the peak period of the scenario, assuming that the transportation assets required during the most stressful portion of the scenario will be adequate to deliver cargoes during less stressful periods. This peak is determined by graphically analyzing the availability and delivery of cargo over time. For our demonstration runs, the peak period corresponds to days C014 to C034. (The *actual* peak period is much smaller, from days C017 to C021; however, we chose the wider period as a precaution to make sure that we captured *all* the peak and took care of any tail effects.)

The scenario considers five types of aircraft (C-5, C-141, C-17, CRAF passenger, and CRAF cargo) and numerous different types of ships that vary by type (e.g., RoRo) and by capacity and speed. For our demonstration runs, we used the five aircraft types but considered only three types of ships—RoRos, bulk, and container. The capacities and speeds of these ship types are averages for the fleet.

Table 6.1

Data Set Characteristics

	Counts for	
	Scenario	Peak Period
Movement requirements	5761	490
Days	181	20
Types of aircraft	5	5
Types of ships	Many	3
Origins	52	3
Destinations	2	2

Finally, we aggregated the original 52 CONUS POEs into three "port aggregates": East Coast, West Coast, and Gulf Coast. (Most current models use this method of aggregation.) There are two PODs (Japan and Korea) in both the original scenario and in our demonstration runs.

Table 6.2 shows the various parameters for the transportation assets shown, which include initially 100 C-5s, 150 C-141s, and various numbers of the three ship types. No more C-5s or C-141s can be obtained, but additional ships can be purchased. The last column in the table shows the 30-year life-cycle costs (LCCs) for these additional ships.

We also assumed that CRAF assets can augment the military airlift fleet. We assumed that 15 long-range, wide-bodied cargo (LRWC) aircraft and 75 long-range, wide-bodied passenger (LRWP) aircraft can be called on (with the resulting cost per aircraft shown in the last column). The model results show how many CRAF assets are needed and when. Finally, we assumed that C-17s can be procured.

The MP model will use the available assets (C-5s, C-141s, ships) first and then will draw upon CRAF assets, C-17s, or additional ships as needed. By examining the model results, we can determine when various asset types are utilized.

MINIMIZING COST AND OTHER TRADE-OFFS

Given the scenario characteristics and the assumptions about transportation assets, we show the results of various types of runs for the MP prototypes. These runs demonstrate the following capabilities:

- Determine the least-cost set of additional transportation assets needed to deliver all cargoes on time.
- Determine the minimum "delay" (i.e., ton-days late), given fixed cargo-availability dates and either only current transportation assets or current assets plus a specified budget to acquire additional transportation assets.
- Determine the minimum early availability of units, given fixed cargo-delivery dates and either only current transportation assets or current transportation assets plus a specified budget to acquire additional transportation assets.

Table 6.2

Transport Resources Available

Asset	On Hand	No. Available	Buy More?	30-yr LCC ($ billion)
C-5	100		No	
C-17	0		Yes	500
C-141	150		No	
LRWC	0	15	No	30
LRWP	0	75	No	15
Bulk	60		Yes	400
Container	40		Yes	400
RoRo	50		Yes	425

- Determine the minimum tons of cargo to preposition, given current transportation assets and fixed cargo-availability and -delivery dates.

Figure 6.1 shows the result of the MP run to minimize the cost of transportation assets to deliver all the cargoes on time, the basic mobility resource requirements question. The y-axis displays the 30-year life-cycle cost of the additional transportation assets. The x-axis shows a measure of lateness; here we want zero lateness, or all cargoes to be delivered by their RDDs. The result shows that, in addition to the available C-5s, C-141s, and ships, we need 290 C-17s, 25 (of the 75) CRAF LRWPs, and all 15 of the CRAF LRWCs. The total 30-year life-cycle cost of these additional assets is approximately $150 billion.

In addition to providing the least-cost set of transportation assets to close the force, the model shows when specific cargoes are loaded at the POEs, what type of asset they travel on, and when they arrive at their destination, similar to the example given in Chapter Three. The model output also provides details on when specific asset types (e.g., the CRAF aircraft) are required and where bottlenecks or slacks exist in the system. Sample output from this first model can be found in Appendix A.

This demonstration shows how the MP prototype can be used to address the basic issue of transportation requirements, a question whose answer cannot be directly obtained with existing models. A solution that requires the purchase of 290 C-17s is impractical. The alternative formulations will allow the analyst to run a series of excursions to trade off one aspect against another in an attempt to arrive at a more realistic solution.

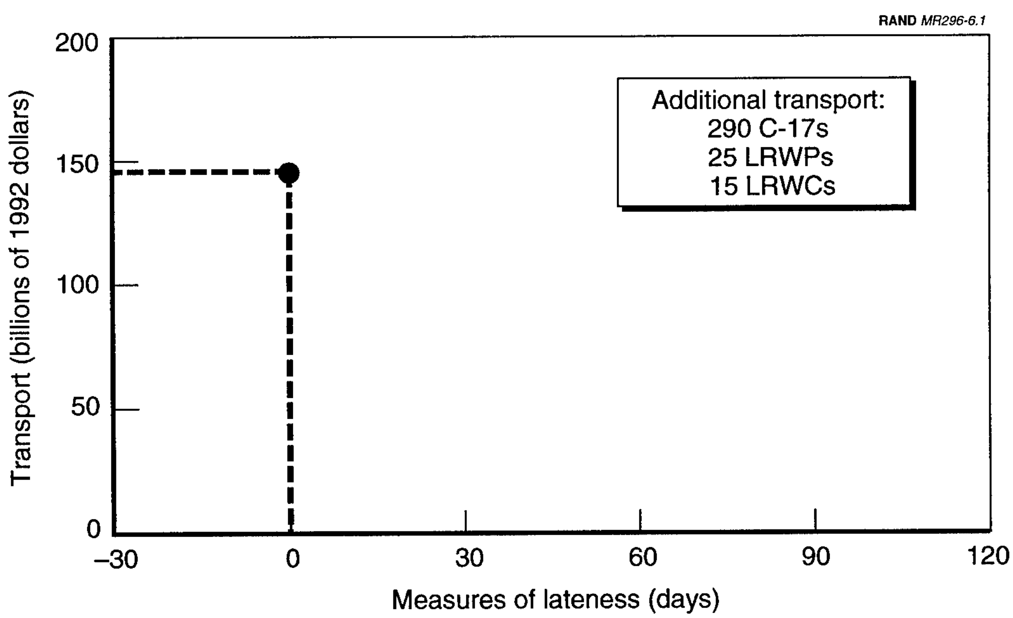

Figure 6.1—Solution for Optimal Assets

Figure 6.2 shows the results of constraining the transportation assets to the available fleet (no additional assets can be procured) and minimizing a measure of lateness. Here, the objective function minimizes ton-days late, with each cargo and each day late valued equally. The model suggests (point **B**) that available transportation assets can deliver the force with approximately 120 thousand ton-days of cargo late. The model also indicates which cargoes are late and when they do arrive.

We could have formulated the model in other ways for these types of questions. For example, we could have adjusted the budget available to buy more assets or we could have attached a cost to lateness and minimized the sum of transportation costs plus lateness costs.

There are also different ways we could have evaluated "lateness." As mentioned, in the run shown we viewed each cargo and each day late as equal. That is, they all had the same penalty cost for being late. We could have put different costs of lateness on different cargoes; for example, we might prefer to have combat units delivered on time at the expense of support units or resupply. Then, we would put a much higher cost on combat unit ton-days late and a lower cost for support units or resupply late. Also, we could have evaluated days late differently. For example, the second day a cargo is late might cost more than the first day late. The model has the flexibility to address various measures of lateness and different lateness costs for different cargoes. Using the MP prototype in this manner provides a measure of optimal capability of the existing transportation fleet.

To extend our previous example further, we conducted two intermediate runs. In one run we specified an additional budget of $50 billion over 30 years and asked the

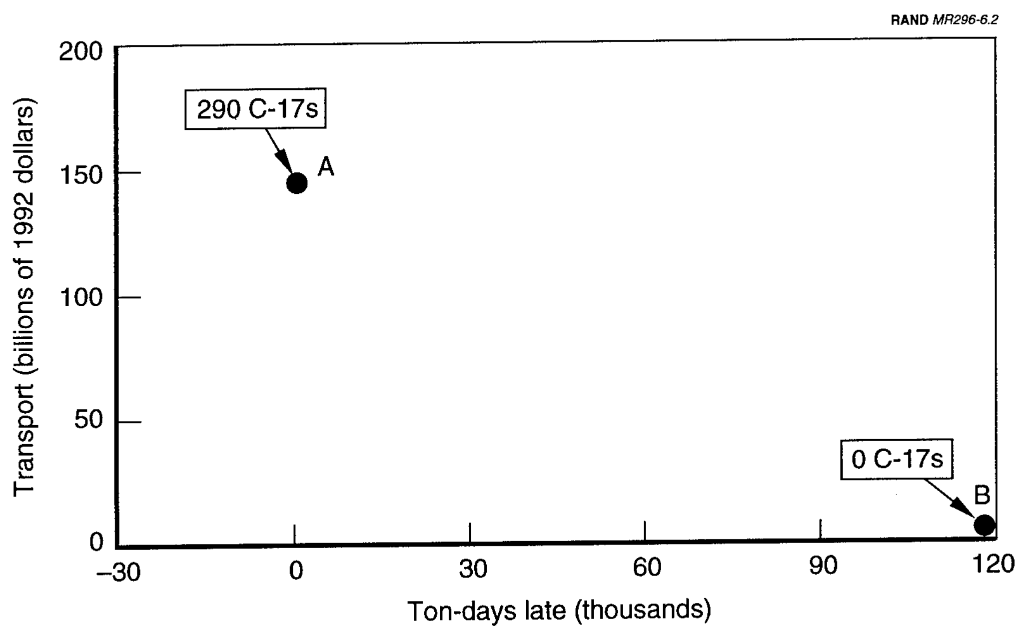

Figure 6.2—Solution for Using Only Available Assets

model to minimize lateness. Ninety-nine C-17s would be procured (in addition to the LRWCs and LRWPs shown before), and the ton-days late would be reduced to approximately 30,000. The second run set an additional budget of $100 billion. As Figure 6.3 indicates, the model suggested that 199 C-17s would be bought and that the cargo would be a total of approximately 5000 ton-days late.

The results of this type of parametric analysis provide insight into the trade-offs between additional transportation assets and lateness. Such a function is convex; that is, there are diseconomies to adding more C-17s. The first increment of $50 billion reduces ton-days late significantly (by 90,000); the second increment of $50 billion has a much smaller effect (further reduction of almost 25,000 ton-days late); the third increment of $50 billion has only a slight effect (reducing lateness from 5000 to 0).

Given this information, decisionmakers can determine whether they are willing to have some cargoes arrive slightly later than planned to reduce transportation costs. Allowing cargoes to arrive later than planned is one solution when transportation assets are limited and budgets are constrained. Each cargo can be viewed as having a time window. If possible, we would like each cargo to depart its POE and arrive at its POD within the time window. Allowing cargoes to arrive late extends one edge of the window.

An alternative approach involves extending the other edge of the time window. One way to think of this option is that it "flattens the peak" by moving some cargo to an earlier time period. That is, excess transportation capacity may be available prior to the peak demand for transportation. If some of the cargo can be moved from a time period when there is more cargo to be moved than there are assets to move that

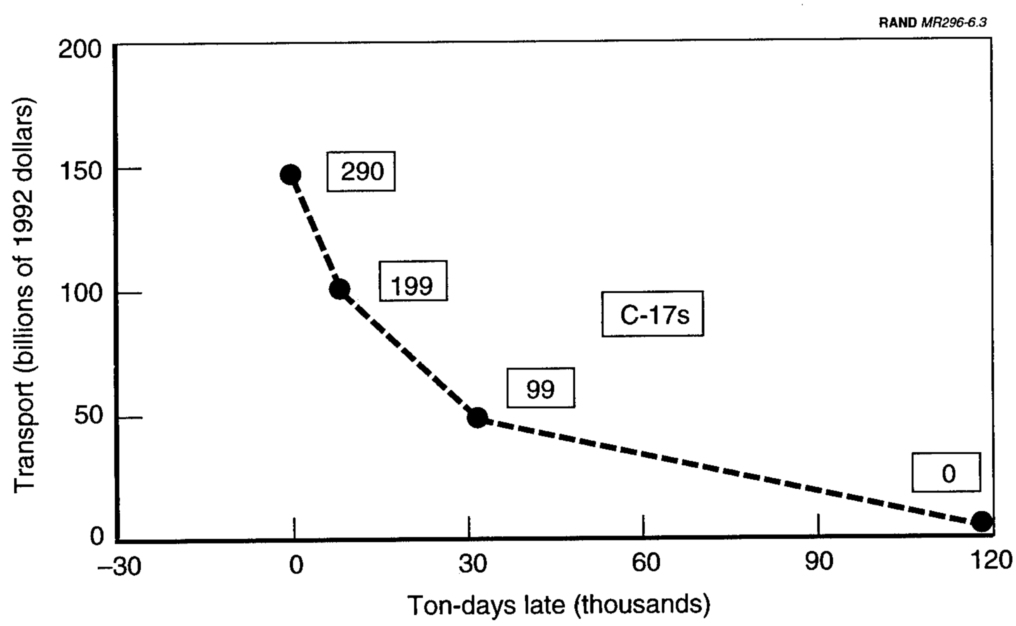

Figure 6.3—Trading Cost for Effectiveness

cargo to an earlier time period when there is excess transport capacity, it may be possible to deliver all cargo on time (although some cargo would arrive in the theater earlier than planned) with available assets.

Figure 6.4 shows the results of various model runs in which the RDDs were fixed and we allowed the units to arrive at the POE earlier than their planned ALDs. We minimized a measure of earliness for existing assets and included an increment of $50 billion to buy additional transportation assets. The top left point is the solution to our basic transportation question—the budget required to deliver all cargoes on time with their ALDs fixed. The point on the lower right is the minimum number of ton-days early (at the POEs) that can be attained with current transportation assets. It suggests that if approximately 90,000 tons-days of cargo could arrive at their POEs earlier, all cargoes could close when planned with current assets. The intermediate point shows the number of ton-days early, given $50 billion to procure additional transportation assets.

As with our example of relaxing RDDs, we can formulate the objective function in a number of different ways and we can cost earliness in different ways (whereas here we valued each day early and each cargo the same). Also, the model run shows which cargoes must be early and when the cargoes must be at their POEs.

With this model formulation, decisionmakers can examine the trade-offs between a unit's readiness (in terms of its ability to depart) and transportation costs. It may be less costly to have certain units available a few days early than to procure the transportation assets necessary to deliver them on time with their planned ALDs. Such analyses may also help in selecting which units should be designated to fill a re-

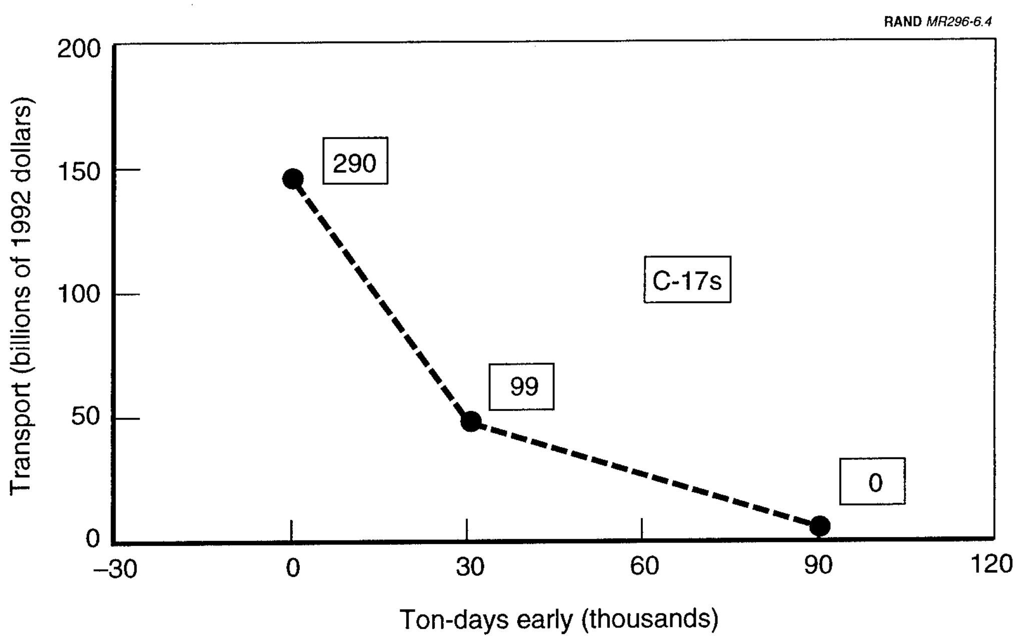

Figure 6.4—Effect of Early Arrivals

quirement. For example, if an armored division is needed in the theater, the analysis of the ALDs may suggest that one armored unit may be preferred over another because it has an ALD that will reduce transportation costs.

Another way to reduce transportation requirements is to preposition some cargoes. In this case, the model minimizes the number of prepositioned tons subject to fixed ALDs, RDDs, and currently available transportation assets. As indicated in Figure 6.5, the model suggests that approximately 30,000 tons of cargo must be prepositioned to close the force when desired, using only existing transportation assets. The model also specifies which cargoes must be prepositioned.

In this example, we have costed all cargoes equally. We could define different prepositioning costs for different cargoes to minimize total prepositioning costs. We could also define the objective function as the sum of transportation costs plus prepositioning costs to find the minimum. In such an example, some cargoes might be prepositioned and some additional transportation assets might be procured.

MULTIPLE SCENARIOS

While exercising our MP prototype, we discovered how easy it is to reformulate the problem to address different issues or to cost different aspects of the overall system. We believe that a production version of the MP prototype will provide the JS/J-4 with an analysis capability that currently does not exist.

The new political order in the world has changed the focus of mobility analyses. Before the upheaval in Eastern Europe and the breakup of the Soviet Union, mobility

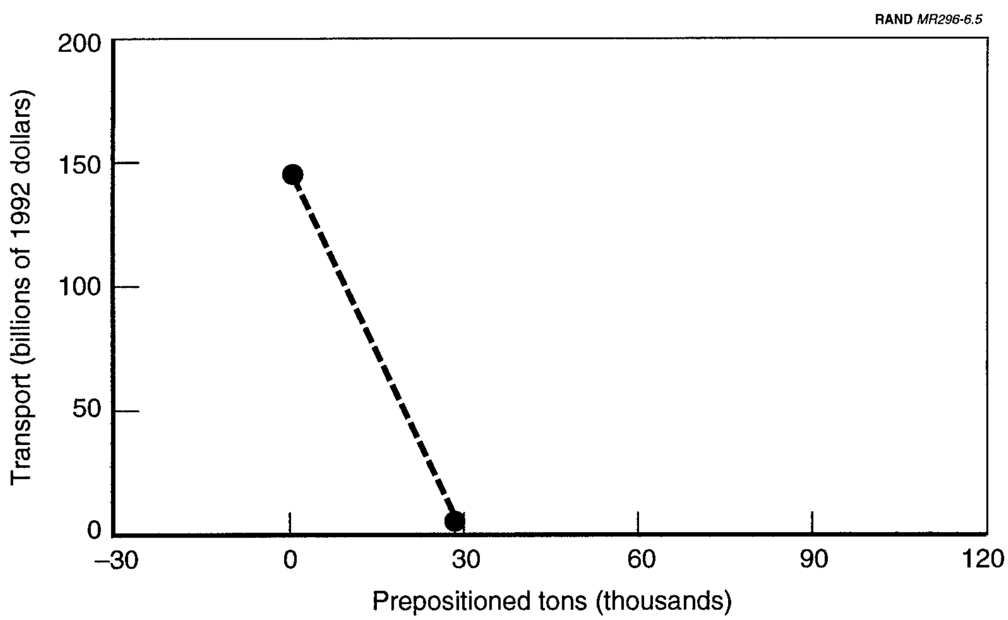

Figure 6.5—Effects of Prepositioning

analysis concentrated on a NATO-versus-Warsaw Pact scenario. Because of U.S. commitments to its allies, the transportation requirements associated with a conventional war in Europe far outweighed the assets needed for other proposed scenarios. Now, however, the mobility community must address numerous scenarios in diverse regions of the world. The various scenarios examined during the Mobility Requirements Study addressed several major and minor scenarios.

These various scenarios tax the mobility system in different ways. Some require primarily airlift assets; others place heavier demands on sealift. Analysts now seek the strategic mobility system that can respond to all potential scenarios. That is, they seek a robust transportation system that, although potentially not optimum for any one case, can satisfy the requirements of many different scenarios. We have been able to demonstrate that the MP prototype can determine the optimal set of transportation assets when looking across several scenarios.

Compared with the counts in Table 6.3, the savings implied by the two accompanying figures may seem exaggerated; however, they illustrate potential savings when alternative scenarios differ greatly in lift requirements but have cargoes that could be shipped by a variety of lift assets, either airlift or sealift. Figures 6.6 and 6.7 illustrate a hypothetical set of scenarios in which scenario 1, if considered alone, would suggest investing mainly in airlift, whereas scenario 2, if considered alone, would suggest investing mainly in sealift, and for which we assume, for the purpose of illustration, that most cargoes can be shipped by either air or sea and are not designated in advance for shipment by a particular transportation mode. Note that we are not assuming that there is enough time to ship by sea, but rather that cargoes have not been preassigned to either airlift or sealift according to their cargo types. This flexibility in assigning movements to vehicles results in the cost savings evidenced in Figures 6.6 and 6.7.

Consider the example shown in Figure 6.6. Scenario 1 places heavy demands on airlift but minor requirements on sealift. This is the scenario examined in the examples above. Scenario 2 makes heavy demands on sealift but fewer on airlift. One solution to the movement problem would be to add the airlift requirements of the first scenario to those of the second, but that approach would overstate the assets required.

By entering both scenarios into the MP prototype at the same time, we can determine the optimal set of transportation assets needed to accomplish either scenario. Table 6.3 shows some characteristics of the two scenarios, both of which were developed by the Joint Staff for the Mobility Requirements Study. To stress our formulation and MP solver, we use these scenarios and a lengthy peak period for both scenarios.

Table 6.4 shows the transport required to deliver all cargoes on time for four different scenarios: 1 only, 2 only, 1 or 2, and 1 and 2. For the 1-and-2 case, the peak period for scenario 2 was programmed to begin 30 days after the beginning of the peak period for scenario 1. Figure 6.7 depicts the result of solving for both scenarios.

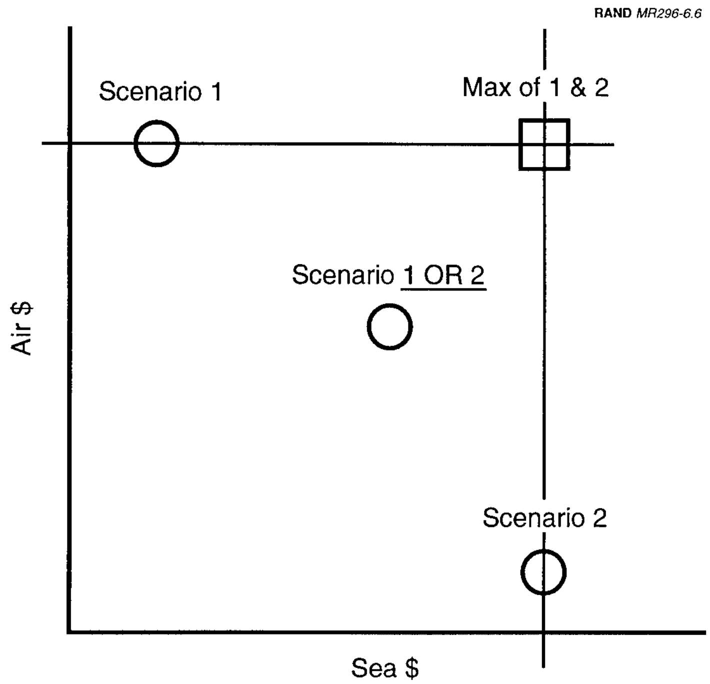

Figure 6.6—Best Set of Assets to Cover Either Scenario 1 or 2

Table 6.3

Data Sets for Sample Scenarios

	Counts			
	Scenario 1		Scenario 2	
Item	Full Scenario	Demo Runs	Full Scenario	Demo Runs
Movement requirements	5761	1258	11,942	8595
Days	181	75	228	75
Types of aircraft	5	5	5	5
Types of ships	Many	3	Many	3
Origins	52	3	66	4
Destinations	2	4	16	2

As noted previously, scenario 1 contains many time-sensitive cargoes and requires the acquisition of a large number of C-17s. These aircraft can then be used, at no additional cost, to deliver most or all the cargoes that would be delivered by ship. A nominal cost for using owned ships and aircraft for the first time induces the model to use an available already-used vehicle before using an available new vehicle. Scenario 2, on the other hand, contains significantly more cargoes but most are less time-sensitive and, consequently, can travel by ship. Note that, although they are not used in the other runs, these ships do not increase the incremental cost because they are already owned by or under lease to the government. The reduction in the number of C-17s that must be purchased *significantly* reduces the incremental costs.

Table 6.4

Transport Required for Two Scenarios
(all cargoes delivered on time)

Vehicle Type	Baseline Available	Vehicles Used by Scenario			
		1	2	1 or 2	1 and 2
Aircraft					
C-5	100	100	100	100	100
C-17	0	290	247	290	290
C-141	150	150	150	150	150
LRWC	15	15	15	15	15
LWRP	75	24	42	42	42
Ships					
Bulk	60		6		
Container	40		40	40	40
RoRo	50		50	50	50
Incremental costs (billion 1992 dollars)		$146	$125	$146	$146

RAND *MR296-6.7*

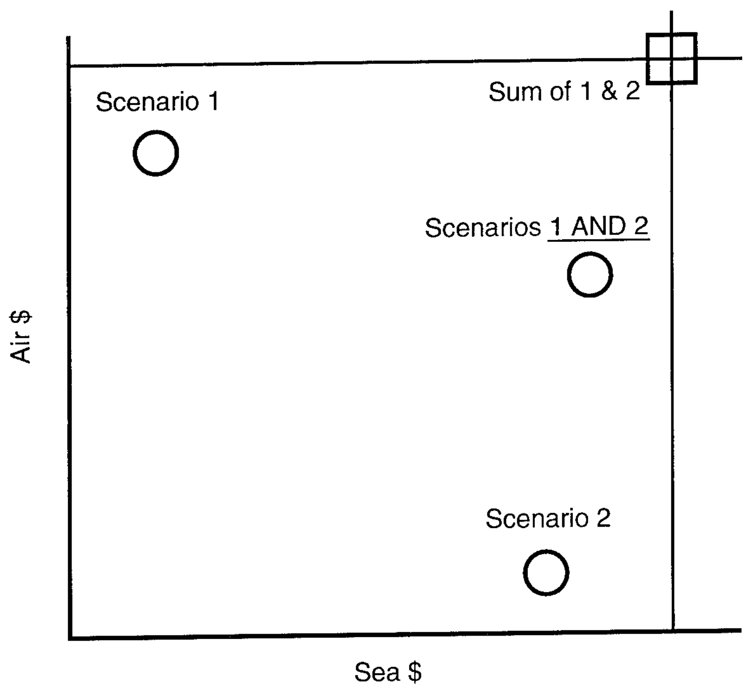

Figure 6.7—Best Set of Assets to Cover Both Scenarios

Table 6.4 shows that, when we enter both scenarios in the model and solve for the vehicle set that most efficiently solves scenario 1 *or* scenario 2, that set, not surprisingly, turns out to be almost the maximum of the two individual sets. All 290 C-17s are still required to deliver all the scenario 1 cargoes on time, and almost all the ships used to service scenario 2 are still needed, despite the additional 43 aircraft.

Finally, the set of vehicles required to deliver all goods on time in the 1-and-2 near-simultaneous case turns out to be the same as that of the either/or case.[1] This result occurs because the peak demands for aircraft in the two scenarios do not overlap. The peak airlift requirement for scenario 1 is sufficiently large that it can carry most of the later routine bulk cargoes for both scenarios.

[1]The MP prototype addresses the near-simultaneous problem by treating the two scenarios as one and computing a least-cost set of assets to handle the total movement requirement.

Chapter Seven

RESULTS, VALIDATION, AND EXTENDING THE PROTOTYPE

RESULTS

Although our original focus when we began the research was on determining the least-cost set of transportation assets to deliver specified forces on time, we have been able to reformulate the basic model to address many other issues of interest. In the preceding chapter, we showed model runs that demonstrate the following capabilities:

- Directly determines the least-cost set of transportation assets to deliver all the forces by their RDDs.
- Determines the minimum amount of lateness, measured in various ways, given an existing set of transportation assets and a limited budget to buy and operate more. The model also shows which cargoes are late and how late they are.
- Examines the trade-offs in having units arrive at POEs earlier than needed and the cost of transportation assets needed to deliver the force on time. The model shows which cargoes require earlier ALDs and how much earlier than planned they must be ready to load.
- Shows how much and which cargoes must be prepositioned to deliver the force on time when transportation assets are limited.
- Examines multiple scenarios simultaneously to determine the robust set of transportation assets needed to meet potential commitments.

We believe that the mobility community has not been able to address these types of issues as effectively with current models. The capabilities offered by a mathematical programming model enhance the set of analysis tools available to the mobility community.

VALIDATION

Models are *validated* to determine whether they accurately represent the real system. Models are also *verified* to ensure that the computer program prepared for the model is correct and is performing properly. The logical internal structure of the mathematical programming formulation and the representation of that structure using the GAMS computer system help us to achieve model verification.[1] Model validation is accomplished through the calibration of the model, usually by comparing model results with real-world output. When discrepancies exist between the model predictions and the actual system's behavior, the model parameters and coefficients are adjusted. This process is repeated until the accuracy of the model is judged acceptable.

It is in model validation that existing simulation models can be interfaced with the mathematical programming model. The mathematical programming formulation assumes the real world operates in an optimal fashion: Cargo is loaded in a way that best utilizes available space, units and equipment arrive when scheduled, and, in general, the system functions in perfect ways. The mathematical programming formulation does not capture those uncertainties that exist in the real system that preclude optimal operations. Simulation models attempt to represent real-world operations, often using Monte Carlo techniques to represent real-world uncertainties.

The least-cost formulation of the mathematical programming model provides the optimal set of transportation assets needed to deliver all cargoes by their required delivery dates. Output from this mathematical programming model can be used as input to an existing simulation model, and the simulated delivery dates can be observed along with other characteristics of the system's operations, such as load factors for various types of aircraft or ships. The factors and parameters of the MP formulation can then be adjusted and the model exercised again. Through this interfacing of the MP and simulation models, the MP model can be calibrated. This calibration will help make the MP model a valid representation of the real system and produce results that do not understate real transportation needs

EXTENDING AND USING THE MP PROTOTYPE

The model we have been using is a prototype, developed to demonstrate the feasibility of underlying principles and to examine alternative formulations. A full production model will require additional effort. Some effort should be expended on constructing the front end of the prototype, such as creating data-entry screens, to assist the analyst in setting up the model runs. Although a prototype, the model does provide immediate capabilities. As analysts become more familiar with it, they will determine ways to enhance it.

We believe the model will offer new capabilities to organizations in the mobility community, most notably the U.S. Transportation Command and the Office of the Assistant Secretary of Defense for Program Analysis and Evaluation.

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[1]The General Algebraic Modeling System (GAMS) is a mathematical programming package developed at the World Bank. It uses its own computer language.

IMPLEMENTATION OF MP MODELS

SIMPLE COST-MINIMIZATION EXAMPLE

Below is the computer program for the simple cost-minimization example described in Chapter Three. It is written in the General Algebraic Modeling System (GAMS) programming language.[1] Following the computer program is a listing of both the linear solution and the integer solution to the program. Insofar as possible, the variable names in the program correspond directly to the variable names used in the mathematical description of the model.

COST-MINIMIZATION PROGRAM CODE: LINEAR SOLUTION

```
set m movement requirements / 1 * 10 / ;
set v lift assets / c141b, c5,  kc10/ ;
set j / bulk, over, pax / ;
table capacity(v,j) capacity of vehicle v for cargo type j
          bulk     over     pax
 c141b    23.0     23.6     153
    c5    69.6     65.0     329
  kc10    62.1     26.4     257
  ;
table tons(m,j)
          bulk     over     pax
     1      15        0       0
     2      17        0       0
     3       0        0     125
     4       0       43      75
     5      71        0      55
     6      21        0      27
     7    37.5        0      25
     8     710        0       0
     9     377        0       0
```

[1]GAMS is a mathematical programming package developed at the World Bank. It uses its own computer language.

```
    10        0       22        0
  ;
parameter L(m,j,v) load factors;
L(m,j,v) = tons(m,j)/capacity(v,j);
parameter a(m) availability of movement m
/
     1       1
     2       1
     3       1
     4       3
     5       4
     6       7
     7       6
     8       7
     9       8
    10      10
 /;
parameter lad(m) due date of movement m
/
     1       2
     2       2
     3       2
     4       5
     5       6
     6      10
     7       9
     8      11
     9      12
    10      13
 /;
set e list of unique poes
/
seattle
st-louis
boston
new-york
san-fran
san-dieg
 /;
set d list of unique pods
/
pingtung
chiayi
taipei
tainan
 /;
parameter poe(m,e) shows which poe shipment m uses
```

```
/
     1.seattle    = 1
     2.seattle    = 1
     3.st-louis    = 1
     4.st-louis    = 1
     5.st-louis    = 1
     6.boston   = 1
     7.new-york    = 1
     8.san-fran   = 1
     9.san-dieg    = 1
    10.san-fran    = 1
 / ;
parameter pod(m,d) shows which pod shipment m uses
/
     1.pingtung    = 1
     2.chiayi    = 1
     3.pingtung    = 1
     4.taipei    = 1
     5.taipei    = 1
     6.tainan    = 1
     7.tainan    = 1
     8.taipei    = 1
     9.pingtung    = 1
    10.pingtung    = 1
 / ;
parameter channel(e,d)
/
      seattle.pingtung = 1
      seattle.chiayi = 1
      st-louis.pingtung = 1
      st-louis.taipei = 1
      boston.tainan = 1
      new-york.tainan = 1
      san-fran.taipei = 1
      san-dieg.pingtung = 1
      san-fran.pingtung = 1
 / ;
parameter b(m,v) latest that m can go on v to meet lad ;
b(m,v) = lad(m) - 1;
set     t /c001  * c013 / ;
parameter tnum(t) number of day t;
        tnum(t) = ord(t);
set     h /c001  * c015 / ;
parameter hnum(h) number of day h;
    hnum(h) = ord(h);
parameter inuse(e,d,t)
/
```

```
     seattle.pingtung.c001 = 1
     seattle.chiayi.c001 = 1
     st-louis.pingtung.c001 = 1
     st-louis.taipei.c003 = 1
     st-louis.taipei.c004 = 1
     st-louis.taipei.c005 = 1
     boston.tainan.c007 = 1
     boston.tainan.c008 = 1
     boston.tainan.c009 = 1
     new-york.tainan.c006 = 1
     new-york.tainan.c007 = 1
     new-york.tainan.c008 = 1
     san-fran.taipei.c007 = 1
     san-fran.taipei.c008 = 1
     san-fran.taipei.c009 = 1
     san-fran.taipei.c010 = 1
     san-dieg.pingtung.c008 = 1
     san-dieg.pingtung.c009 = 1
     san-dieg.pingtung.c010 = 1
     san-dieg.pingtung.c011 = 1
     san-fran.pingtung.c010 = 1
     san-fran.pingtung.c011 = 1
     san-fran.pingtung.c012 = 1
 / ;
parameter dollars(v) guesstimate cost of additional lift assets of type v
        /
        c141b    1
        c5       4
        kc10     2
        / ;
parameter S(e,d,v) cycle time;
S(e,d,v) = 1;
parameter N(v) number of lift assets of type v at on hand
        /
        c141b    1
        c5       1
        kc10     1
        / ;
variables
        U(e, d, t, v) no. of v loaded at POE e in t bound for POD d
        X(m, j, t, v) no. of v loaded with j of m on day t
        Y(v) number of type v lift assets to be acquired
        z cost to be minimized
;
positive variables X, U, Y;
equations
```

```
     cost define objective function
     cost2 dummy constraint
     demand(m,j) every movement is shipped in its entirety
     supply1(e,d,t,v) supply of shipping
     supply2(h,v) supply of shipping;
cost .. z =e= sum(v, dollars(v)*Y(v)) ;
demand(m,j) $ (tons(m,j)).. sum( (v,t) $
(tnum(t) ge A(m) and tnum(t) le B(m,v)),
  (1/L(m,j,v))*X(m,j,t,v) ) =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
 .. sum( (m,j)$ (poe(m,e) and pod(m,d) and
(tnum(t) ge A(m) and tnum(t) le B(m,v))), X(m,j,t,v) ) - U(e,d,t,v)  =l= 0 ;
supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
and inuse(e,d,t)), U(e,d,t,v) )
                - Y(v) =l= N(v) ;
model smmIP / demand, supply1, supply2, cost /;
option lp=minos5;
solve smmIP using lp minimizing z ;
```

COST-MINIMIZATION RESULTS: LINEAR PROGRAM

```
---- DEMAND      =E=  every movement is shipped in its entirety
DEMAND(1,BULK)..  1.5333*X(1,BULK,C001,C141B) + 4.64*X(1,BULK,C001,C5)
      + 4.14*X(1,BULK,C001,KC10) =E= 1 ; (LHS = 0 ***)
DEMAND(2,BULK)..  1.3529*X(2,BULK,C001,C141B) + 4.0941*X(2,BULK,C001,C5)
      + 3.6529*X(2,BULK,C001,KC10) =E= 1 ; (LHS = 0 ***)
DEMAND(3,PAX)..  1.224*X(3,PAX,C001,C141B) + 2.632*X(3,PAX,C001,C5)
      + 2.056*X(3,PAX,C001,KC10) =E= 1 ; (LHS = 0 ***)
REMAINING 11 ENTRIES SKIPPED
---- SUPPLY1     =L=  supply of shipping
SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)..  - U(SEATTLE,PINGTUNG,C001,C141B)
      + X(1,BULK,C001,C141B) + X(1,OVER,C001,C141B) + X(1,PAX,C001,C141B)
      =L= 0 ; (LHS = 0)
SUPPLY1(SEATTLE,PINGTUNG,C001,C5)..  - U(SEATTLE,PINGTUNG,C001,C5)
      + X(1,BULK,C001,C5) + X(1,OVER,C001,C5) + X(1,PAX,C001,C5) =L= 0 ;
      (LHS = 0)
SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)..  - U(SEATTLE,PINGTUNG,C001,KC10)
      + X(1,BULK,C001,KC10) + X(1,OVER,C001,KC10) + X(1,PAX,C001,KC10) =L= 0 ;
      (LHS = 0)
REMAINING 66 ENTRIES SKIPPED
---- SUPPLY2     =L=  supply of shipping
SUPPLY2(C001,C141B)..  U(SEATTLE,PINGTUNG,C001,C141B)
      + U(SEATTLE,CHIAYI,C001,C141B) + U(ST-LOUIS,PINGTUNG,C001,C141B)
      - Y(C141B) =L= 1 ; (LHS = 0)
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SUPPLY2(C001,C5)..  U(SEATTLE,PINGTUNG,C001,C5) + U(SEATTLE,CHIAYI,C001,C5)
      + U(ST-LOUIS,PINGTUNG,C001,C5) - Y(C5) =L= 1 ; (LHS = 0)
     SUPPLY2      =L=  supply of shipping
SUPPLY2(C001,KC10)..  U(SEATTLE,PINGTUNG,C001,KC10)
      + U(SEATTLE,CHIAYI,C001,KC10) + U(ST-LOUIS,PINGTUNG,C001,KC10)
      - Y(KC10) =L= 1 ; (LHS = 0)
REMAINING 42 ENTRIES SKIPPED
---- COST         =E=  define objective function
COST..  - Y(C141B) - 4*Y(C5) - 2*Y(KC10) + Z =E= 0 ; (LHS = 0)
---- U           no. of v loaded at POE e in t bound for POD d
U(SEATTLE,PINGTUNG,C001,C141B)
                (.LO, .L, .UP = 0, 0, +INF)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)
        1        SUPPLY2(C001,C141B)
        1        SUPPLY2(C002,C141B)
U(SEATTLE,PINGTUNG,C001,C5)
                (.LO, .L, .UP = 0, 0, +INF)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,C5)
        1        SUPPLY2(C001,C5)
        1        SUPPLY2(C002,C5)
U(SEATTLE,PINGTUNG,C001,KC10)
                (.LO, .L, .UP = 0, 0, +INF)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)
        1        SUPPLY2(C001,KC10)
        1        SUPPLY2(C002,KC10)
REMAINING 66 ENTRIES SKIPPED
---- X           no. of v loaded with j of m on day t
X(1,BULK,C001,C141B)
                (.LO, .L, .UP = 0, 0, +INF)
        1.5333   DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)
X(1,BULK,C001,C5)
                (.LO, .L, .UP = 0, 0, +INF)
        4.64     DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,C5)
X(1,BULK,C001,KC10)
                (.LO, .L, .UP = 0, 0, +INF)
        4.14     DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)
REMAINING 213 ENTRIES SKIPPED
---- Y           number of type v lift assets to be acquired
Y(C141B)
                (.LO, .L, .UP = 0, 0, +INF)
       -1        SUPPLY2(C001,C141B)
       -1        SUPPLY2(C002,C141B)
       -1        SUPPLY2(C003,C141B)
       -1        SUPPLY2(C004,C141B)
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        -1      SUPPLY2(C005,C141B)
        -1      SUPPLY2(C006,C141B)
        -1      SUPPLY2(C007,C141B)
        -1      SUPPLY2(C008,C141B)
        -1      SUPPLY2(C009,C141B)
        -1      SUPPLY2(C010,C141B)
        -1      SUPPLY2(C011,C141B)
        -1      SUPPLY2(C012,C141B)
        -1      SUPPLY2(C013,C141B)
        -1      SUPPLY2(C014,C141B)
        -1      SUPPLY2(C015,C141B)
        -1      COST
Y(C5)
                (.LO, .L, .UP = 0, 0, +INF)
        -1      SUPPLY2(C001,C5)
        -1      SUPPLY2(C002,C5)
        -1      SUPPLY2(C003,C5)
        -1      SUPPLY2(C004,C5)
        -1      SUPPLY2(C005,C5)
        -1      SUPPLY2(C006,C5)
        -1      SUPPLY2(C007,C5)
        -1      SUPPLY2(C008,C5)
        -1      SUPPLY2(C009,C5)
        -1      SUPPLY2(C010,C5)
        -1      SUPPLY2(C011,C5)
        -1      SUPPLY2(C012,C5)
        -1      SUPPLY2(C013,C5)
        -1      SUPPLY2(C014,C5)
        -1      SUPPLY2(C015,C5)
        -4      COST
Y(KC10)
                (.LO, .L, .UP = 0, 0, +INF)
        -1      SUPPLY2(C001,KC10)
        -1      SUPPLY2(C002,KC10)
        -1      SUPPLY2(C003,KC10)
        -1      SUPPLY2(C004,KC10)
        -1      SUPPLY2(C005,KC10)
        -1      SUPPLY2(C006,KC10)
        -1      SUPPLY2(C007,KC10)
        -1      SUPPLY2(C008,KC10)
        -1      SUPPLY2(C009,KC10)
        -1      SUPPLY2(C010,KC10)
        -1      SUPPLY2(C011,KC10)
        -1      SUPPLY2(C012,KC10)
        -1      SUPPLY2(C013,KC10)
        -1      SUPPLY2(C014,KC10)
        -1      SUPPLY2(C015,KC10)
        -2      COST
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---- Z          cost to be minimized
Z
                (.LO, .L, .UP = -INF, 0, +INF)
        1       COST
---- EQU DEMAND      every movement is shipped in its entirety
           LOWER      LEVEL      UPPER     MARGINAL
1 .BULK     1.000      1.000      1.000      EPS
2 .BULK     1.000      1.000      1.000      EPS
3 .PAX      1.000      1.000      1.000      EPS
4 .OVER     1.000      1.000      1.000      EPS
4 .PAX      1.000      1.000      1.000      EPS
5 .BULK     1.000      1.000      1.000      EPS
5 .PAX      1.000      1.000      1.000      EPS
6 .BULK     1.000      1.000      1.000     0.225
6 .PAX      1.000      1.000      1.000     0.044
7 .BULK     1.000      1.000      1.000     0.403
7 .PAX      1.000      1.000      1.000     0.040
8 .BULK     1.000      1.000      1.000     7.622
9 .BULK     1.000      1.000      1.000     4.047
10.OVER     1.000      1.000      1.000      EPS
---- EQU SUPPLY1     supply of shipping
                                  LOWER      LEVEL      UPPER     MARGINAL
SEATTLE .PINGTUNG.C001.C141B       -INF        .          .          EPS
SEATTLE .PINGTUNG.C001.C5          -INF        .          .          EPS
SEATTLE .PINGTUNG.C001.KC10        -INF        .          .          EPS
SEATTLE .CHIAYI  .C001.C141B       -INF        .          .          EPS
SEATTLE .CHIAYI  .C001.C5          -INF        .          .          EPS
SEATTLE .CHIAYI  .C001.KC10        -INF        .          .          EPS
ST-LOUIS.PINGTUNG.C001.C141B       -INF        .          .          EPS
ST-LOUIS.PINGTUNG.C001.C5          -INF        .          .          EPS
ST-LOUIS.PINGTUNG.C001.KC10        -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C003.C141B       -INF        .          .           .
ST-LOUIS.TAIPEI  .C003.C5          -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C003.KC10        -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C004.C141B       -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C004.C5          -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C004.KC10        -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C005.C141B       -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C005.C5          -INF        .          .          EPS
ST-LOUIS.TAIPEI  .C005.KC10        -INF        .          .          EPS
BOSTON  .TAINAN  .C007.C141B       -INF        .          .        -0.247
BOSTON  .TAINAN  .C007.C5          -INF        .          .        -0.747
BOSTON  .TAINAN  .C007.KC10        -INF        .          .        -0.667
BOSTON  .TAINAN  .C008.C141B       -INF        .          .        -0.247
BOSTON  .TAINAN  .C008.C5          -INF        .          .        -0.747
BOSTON  .TAINAN  .C008.KC10        -INF        .          .        -0.667
BOSTON  .TAINAN  .C009.C141B       -INF        .          .        -0.247
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BOSTON   .TAINAN   .C009.C5         -INF         .          .          -0.747
BOSTON   .TAINAN   .C009.KC10       -INF         .          .          -0.667
NEW-YORK.TAINAN    .C006.C141B      -INF         .          .          -0.247
NEW-YORK.TAINAN    .C006.C5         -INF         .          .          -0.747
NEW-YORK.TAINAN    .C006.KC10       -INF         .          .          -0.667
NEW-YORK.TAINAN    .C007.C141B      -INF         .          .          -0.247
NEW-YORK.TAINAN    .C007.C5         -INF         .          .          -0.747
NEW-YORK.TAINAN    .C007.KC10       -INF         .          .          -0.667
NEW-YORK.TAINAN    .C008.C141B      -INF         .          .          -0.247
NEW-YORK.TAINAN    .C008.C5         -INF         .          .          -0.747
NEW-YORK.TAINAN    .C008.KC10       -INF         .          .          -0.667
SAN-FRAN.PINGTUNG.C010.C141B        -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C010.C5           -INF         .          .          -0.747
SAN-FRAN.PINGTUNG.C010.KC10         -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C011.C141B        -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C011.C5           -INF         .          .          -0.747
SAN-FRAN.PINGTUNG.C011.KC10         -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C012.C141B        -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C012.C5           -INF         .          .            EPS
SAN-FRAN.PINGTUNG.C012.KC10         -INF         .          .            EPS
SAN-FRAN.TAIPEI    .C007.C141B      -INF         .          .          -0.247
SAN-FRAN.TAIPEI    .C007.C5         -INF         .          .          -0.747
SAN-FRAN.TAIPEI    .C007.KC10       -INF         .          .          -0.667
SAN-FRAN.TAIPEI    .C008.C141B      -INF         .          .          -0.247
SAN-FRAN.TAIPEI    .C008.C5         -INF         .          .          -0.747
SAN-FRAN.TAIPEI    .C008.KC10       -INF         .          .          -0.667
SAN-FRAN.TAIPEI    .C009.C141B      -INF         .          .          -0.247
SAN-FRAN.TAIPEI    .C009.C5         -INF         .          .          -0.747
SAN-FRAN.TAIPEI    .C009.KC10       -INF         .          .          -0.667
SAN-FRAN.TAIPEI    .C010.C141B      -INF         .          .          -0.247
SAN-FRAN.TAIPEI    .C010.C5         -INF         .          .          -0.747
SAN-FRAN.TAIPEI    .C010.KC10       -INF         .          .          -0.667
SAN-DIEG.PINGTUNG.C008.C141B        -INF         .          .          -0.247
SAN-DIEG.PINGTUNG.C008.C5           -INF         .          .          -0.747
SAN-DIEG.PINGTUNG.C008.KC10         -INF         .          .          -0.667
SAN-DIEG.PINGTUNG.C009.C141B        -INF         .          .          -0.247
SAN-DIEG.PINGTUNG.C009.C5           -INF         .          .          -0.747
SAN-DIEG.PINGTUNG.C009.KC10         -INF         .          .          -0.667
SAN-DIEG.PINGTUNG.C010.C141B        -INF         .          .          -0.247
SAN-DIEG.PINGTUNG.C010.C5           -INF         .          .          -0.747
SAN-DIEG.PINGTUNG.C010.KC10         -INF         .          .          -0.667
SAN-DIEG.PINGTUNG.C011.C141B        -INF         .          .          -0.247
SAN-DIEG.PINGTUNG.C011.C5           -INF         .          .          -0.747
SAN-DIEG.PINGTUNG.C011.KC10         -INF         .          .          -0.667
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---- EQU SUPPLY2      supply of shipping

               LOWER     LEVEL     UPPER    MARGINAL

C001.C141B     -INF        .       1.000       .
C001.C5        -INF      0.840     1.000       .
C001.KC10      -INF     -3.700     1.000       .
C002.C141B     -INF        .       1.000       .
C002.C5        -INF      0.840     1.000       .
C002.KC10      -INF     -3.700     1.000       .
C003.C141B     -INF        .       1.000       .
C003.C5        -INF        .       1.000       .
C003.KC10      -INF     -0.649     1.000       .
C004.C141B     -INF        .       1.000       .
C004.C5        -INF        .       1.000       .
C004.KC10      -INF      1.000     1.000      EPS
C005.C141B     -INF        .       1.000       .
C005.C5        -INF        .       1.000       .
C005.KC10      -INF     -2.050     1.000       .
C006.C141B     -INF      0.163     1.000       .
C006.C5        -INF      0.539     1.000       .
C006.KC10      -INF     -3.700     1.000       .
C007.C141B     -INF      1.000     1.000    -0.247
C007.C5        -INF      1.000     1.000    -0.747
C007.KC10      -INF      1.000     1.000    -0.667
C008.C141B     -INF      1.000     1.000      EPS
C008.C5        -INF      1.000     1.000      EPS
C008.KC10      -INF      1.000     1.000      EPS
C009.C141B     -INF      1.000     1.000    -0.247
C009.C5        -INF      1.000     1.000    -0.747
C009.KC10      -INF      1.000     1.000    -0.667
C010.C141B     -INF      1.000     1.000      EPS
C010.C5        -INF      1.000     1.000      EPS
C010.KC10      -INF      1.000     1.000      EPS
C011.C141B     -INF      1.000     1.000    -0.247
C011.C5        -INF      1.000     1.000    -0.747
C011.KC10      -INF      1.000     1.000    -0.667
C012.C141B     -INF      0.837     1.000       .
C012.C5        -INF      0.800     1.000       .
C012.KC10      -INF      1.000     1.000       .
C013.C141B     -INF        .       1.000       .
C013.C5        -INF      0.338     1.000       .
C013.KC10      -INF     -3.700     1.000       .
C014.C141B     -INF        .       1.000       .
C014.C5        -INF        .       1.000       .
C014.KC10      -INF     -3.700     1.000       .
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C015.C141B      -INF        .         1.000        .
C015.C5         -INF        .         1.000        .
C015.KC10       -INF     -3.700       1.000        .
                         LOWER       LEVEL       UPPER      MARGINAL
---- EQU COST              .           .           .          1.000
  COST        define objective function
---- VAR U             no. of v loaded at POE e in t bound for POD d
                                     LOWER       LEVEL       UPPER     MARGINAL
SEATTLE .PINGTUNG.C001.C141B           .           .          +INF        EPS
SEATTLE .PINGTUNG.C001.C5              .         0.216        +INF         .
SEATTLE .PINGTUNG.C001.KC10            .           .          +INF        EPS
SEATTLE .CHIAYI  .C001.C141B           .           .          +INF        EPS
SEATTLE .CHIAYI  .C001.C5              .         0.244        +INF         .
SEATTLE .CHIAYI  .C001.KC10            .           .          +INF        EPS
ST-LOUIS.PINGTUNG.C001.C141B           .           .          +INF        EPS
ST-LOUIS.PINGTUNG.C001.C5              .         0.380        +INF         .
ST-LOUIS.PINGTUNG.C001.KC10            .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C003.C141B           .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C003.C5              .           .          +INF         .
ST-LOUIS.TAIPEI  .C003.KC10            .         3.050        +INF         .
ST-LOUIS.TAIPEI  .C004.C141B           .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C004.C5              .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C004.KC10            .         1.649        +INF         .
ST-LOUIS.TAIPEI  .C005.C141B           .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C005.C5              .           .          +INF        EPS
ST-LOUIS.TAIPEI  .C005.KC10            .           .          +INF        EPS
BOSTON  .TAINAN  .C007.C141B           .         0.176        +INF         .
BOSTON  .TAINAN  .C007.C5              .           .          +INF         .
BOSTON  .TAINAN  .C007.KC10            .           .          +INF        EPS
BOSTON  .TAINAN  .C008.C141B           .           .          +INF        EPS
BOSTON  .TAINAN  .C008.C5              .         0.302        +INF         .
BOSTON  .TAINAN  .C008.KC10            .           .          +INF        EPS
BOSTON  .TAINAN  .C009.C141B           .           .          +INF        EPS
BOSTON  .TAINAN  .C009.C5              .           .          +INF         .
BOSTON  .TAINAN  .C009.KC10            .           .          +INF         .
NEW-YORK.TAINAN  .C006.C141B           .         0.163        +INF         .
NEW-YORK.TAINAN  .C006.C5              .         0.539        +INF         .
NEW-YORK.TAINAN  .C006.KC10            .           .          +INF         .
NEW-YORK.TAINAN  .C007.C141B           .           .          +INF        EPS
NEW-YORK.TAINAN  .C007.C5              .           .          +INF        EPS
NEW-YORK.TAINAN  .C007.KC10            .           .          +INF        EPS
NEW-YORK.TAINAN  .C008.C141B           .           .          +INF        EPS
NEW-YORK.TAINAN  .C008.C5              .           .          +INF        EPS
NEW-YORK.TAINAN  .C008.KC10            .           .          +INF        EPS
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SAN-FRAN.PINGTUNG.C010.C141B	.	.	+INF	0.247
SAN-FRAN.PINGTUNG.C010.C5	.	.	+INF	.
SAN-FRAN.PINGTUNG.C010.KC10	.	.	+INF	0.667
SAN-FRAN.PINGTUNG.C011.C141B	.	.	+INF	0.247
SAN-FRAN.PINGTUNG.C011.C5	.	.	+INF	.
SAN-FRAN.PINGTUNG.C011.KC10	.	.	+INF	0.667
SAN-FRAN.PINGTUNG.C012.C141B	.	.	+INF	EPS
SAN-FRAN.PINGTUNG.C012.C5	.	0.338	+INF	.
SAN-FRAN.PINGTUNG.C012.KC10	.	.	+INF	EPS
SAN-FRAN.TAIPEI .C007.C141B	.	0.660	+INF	.
SAN-FRAN.TAIPEI .C007.C5	.	0.461	+INF	.
SAN-FRAN.TAIPEI .C007.KC10	.	4.700	+INF	.
SAN-FRAN.TAIPEI .C008.C141B	.	0.163	+INF	.
SAN-FRAN.TAIPEI .C008.C5	.	0.237	+INF	.
SAN-FRAN.TAIPEI .C008.KC10	.	.	+INF	.
SAN-FRAN.TAIPEI .C009.C141B	.	.	+INF	EPS
SAN-FRAN.TAIPEI .C009.C5	.	0.306	+INF	.
SAN-FRAN.TAIPEI .C009.KC10	.	4.700	+INF	.
SAN-FRAN.TAIPEI .C010.C141B	.	.	+INF	EPS
SAN-FRAN.TAIPEI .C010.C5	.	0.539	+INF	.
SAN-FRAN.TAIPEI .C010.KC10	.	.	+INF	.
SAN-DIEG.PINGTUNG.C008.C141B	.	.	+INF	EPS
SAN-DIEG.PINGTUNG.C008.C5	.	.	+INF	.
SAN-DIEG.PINGTUNG.C008.KC10	.	.	+INF	.
SAN-DIEG.PINGTUNG.C009.C141B	.	0.837	+INF	.
SAN-DIEG.PINGTUNG.C009.C5	.	0.155	+INF	.
SAN-DIEG.PINGTUNG.C009.KC10	.	.	+INF	EPS
SAN-DIEG.PINGTUNG.C010.C141B	.	0.163	+INF	.
SAN-DIEG.PINGTUNG.C010.C5	.	.	+INF	.
SAN-DIEG.PINGTUNG.C010.KC10	.	.	+INF	EPS
SAN-DIEG.PINGTUNG.C011.C141B	.	0.837	+INF	.
SAN-DIEG.PINGTUNG.C011.C5	.	0.461	+INF	.
SAN-DIEG.PINGTUNG.C011.KC10	.	4.700	+INF	.

---- VAR X no. of v loaded with j of m on day t

	LOWER	LEVEL	UPPER	MARGINAL
1 .BULK.C001.C141B	.	.	+INF	.
1 .BULK.C001.C5	.	0.216	+INF	.
1 .BULK.C001.KC10	.	.	+INF	EPS
1 .OVER.C001.C141B	.	.	+INF	EPS
1 .OVER.C001.C5	.	.	+INF	EPS
1 .OVER.C001.KC10	.	.	+INF	.
1 .PAX .C001.C141B	.	.	+INF	EPS
1 .PAX .C001.C5	.	.	+INF	EPS
1 .PAX .C001.KC10	.	.	+INF	EPS

2 .BULK.C001.C141B	.	.	+INF	.
2 .BULK.C001.C5	.	0.244	+INF	.
2 .BULK.C001.KC10	.	.	+INF	EPS
2 .OVER.C001.C141B	.	.	+INF	EPS
2 .OVER.C001.C5	.	.	+INF	EPS
2 .OVER.C001.KC10	.	.	+INF	.
2 .PAX .C001.C141B	.	.	+INF	EPS
2 .PAX .C001.C5	.	.	+INF	EPS
2 .PAX .C001.KC10	.	.	+INF	EPS
3 .BULK.C001.C141B	.	.	+INF	EPS
3 .BULK.C001.C5	.	.	+INF	EPS
3 .BULK.C001.KC10	.	.	+INF	EPS
3 .OVER.C001.C141B	.	.	+INF	EPS
3 .OVER.C001.C5	.	.	+INF	EPS
3 .OVER.C001.KC10	.	.	+INF	EPS
3 .PAX .C001.C141B	.	.	+INF	.
3 .PAX .C001.C5	.	0.380	+INF	.
3 .PAX .C001.KC10	.	.	+INF	.
4 .BULK.C003.C141B	.	.	+INF	EPS
4 .BULK.C003.C5	.	.	+INF	EPS
4 .BULK.C003.KC10	.	1.422	+INF	.
4 .BULK.C004.C141B	.	.	+INF	EPS
4 .BULK.C004.C5	.	.	+INF	EPS
4 .BULK.C004.KC10	.	.	+INF	EPS
4 .OVER.C003.C141B	.	.	+INF	EPS
4 .OVER.C003.C5	.	.	+INF	EPS
4 .OVER.C003.KC10	.	1.629	+INF	.
4 .OVER.C004.C141B	.	.	+INF	EPS
4 .OVER.C004.C5	.	.	+INF	EPS
4 .OVER.C004.KC10	.	.	+INF	EPS
4 .PAX .C003.C141B	.	.	+INF	EPS
4 .PAX .C003.C5	.	.	+INF	EPS
4 .PAX .C003.KC10	.	.	+INF	EPS
4 .PAX .C004.C141B	.	.	+INF	EPS
4 .PAX .C004.C5	.	.	+INF	EPS
4 .PAX .C004.KC10	.	0.292	+INF	.
5 .BULK.C004.C141B	.	.	+INF	EPS
5 .BULK.C004.C5	.	.	+INF	EPS
5 .BULK.C004.KC10	.	1.143	+INF	.
5 .BULK.C005.C141B	.	.	+INF	EPS
5 .BULK.C005.C5	.	.	+INF	EPS
5 .BULK.C005.KC10	.	.	+INF	EPS
5 .OVER.C004.C141B	.	.	+INF	EPS
5 .OVER.C004.C5	.	.	+INF	.
5 .OVER.C004.KC10	.	.	+INF	EPS
5 .OVER.C005.C141B	.	.	+INF	.

5 .OVER.C005.C5	.	.	+INF	EPS
5 .OVER.C005.KC10	.	.	+INF	EPS
5 .PAX .C004.C141B	.	.	+INF	.
5 .PAX .C004.C5	.	.	+INF	EPS
5 .PAX .C004.KC10	.	0.214	+INF	.
5 .PAX .C005.C141B	.	.	+INF	EPS
5 .PAX .C005.C5	.	.	+INF	.
5 .PAX .C005.KC10	.	.	+INF	.
6 .BULK.C007.C141B	.	.	+INF	EPS
6 .BULK.C007.C5	.	.	+INF	EPS
6 .BULK.C007.KC10	.	.	+INF	.
6 .BULK.C008.C141B	.	.	+INF	.
6 .BULK.C008.C5	.	0.302	+INF	.
6 .BULK.C008.KC10	.	.	+INF	.
6 .BULK.C009.C141B	.	.	+INF	.
6 .BULK.C009.C5	.	.	+INF	EPS
6 .BULK.C009.KC10	.	.	+INF	EPS
6 .OVER.C007.C141B	.	.	+INF	0.247
6 .OVER.C007.C5	.	.	+INF	0.747
6 .OVER.C007.KC10	.	.	+INF	0.667
6 .OVER.C008.C141B	.	.	+INF	0.247
6 .OVER.C008.C5	.	.	+INF	0.747
6 .OVER.C008.KC10	.	.	+INF	0.667
6 .OVER.C009.C141B	.	.	+INF	0.247
6 .OVER.C009.C5	.	.	+INF	0.747
6 .OVER.C009.KC10	.	.	+INF	0.667
6 .PAX .C007.C141B	.	0.176	+INF	.
6 .PAX .C007.C5	.	.	+INF	0.216
6 .PAX .C007.KC10	.	.	+INF	0.252
6 .PAX .C008.C141B	.	.	+INF	EPS
6 .PAX .C008.C5	.	.	+INF	0.216
6 .PAX .C008.KC10	.	.	+INF	0.252
6 .PAX .C009.C141B	.	.	+INF	EPS
6 .PAX .C009.C5	.	.	+INF	0.216
6 .PAX .C009.KC10	.	.	+INF	0.252
7 .BULK.C006.C141B	.	.	+INF	EPS
7 .BULK.C006.C5	.	0.539	+INF	.
7 .BULK.C006.KC10	.	.	+INF	.
7 .BULK.C007.C141B	.	.	+INF	.
7 .BULK.C007.C5	.	.	+INF	.
7 .BULK.C007.KC10	.	.	+INF	.
7 .BULK.C008.C141B	.	.	+INF	.
7 .BULK.C008.C5	.	.	+INF	.
7 .BULK.C008.KC10	.	.	+INF	.
7 .OVER.C006.C141B	.	.	+INF	0.247
7 .OVER.C006.C5	.	.	+INF	0.747

7 .OVER.C006.KC10	.	.	+INF	0.667
7 .OVER.C007.C141B	.	.	+INF	0.247
7 .OVER.C007.C5	.	.	+INF	0.747
7 .OVER.C007.KC10	.	.	+INF	0.667
7 .OVER.C008.C141B	.	.	+INF	0.247
7 .OVER.C008.C5	.	.	+INF	0.747
7 .OVER.C008.KC10	.	.	+INF	0.667
7 .PAX .C006.C141B	.	0.163	+INF	.
7 .PAX .C006.C5	.	.	+INF	0.216
7 .PAX .C006.KC10	.	.	+INF	0.252
7 .PAX .C007.C141B	.	.	+INF	EPS
7 .PAX .C007.C5	.	.	+INF	0.216
7 .PAX .C007.KC10	.	.	+INF	0.252
7 .PAX .C008.C141B	.	.	+INF	EPS
7 .PAX .C008.C5	.	.	+INF	0.216
7 .PAX .C008.KC10	.	.	+INF	0.252
8 .BULK.C007.C141B	.	0.660	+INF	.
8 .BULK.C007.C5	.	0.461	+INF	.
8 .BULK.C007.KC10	.	4.700	+INF	.
8 .BULK.C008.C141B	.	0.163	+INF	.
8 .BULK.C008.C5	.	0.237	+INF	.
8 .BULK.C008.KC10	.	.	+INF	.
8 .BULK.C009.C141B	.	.	+INF	.
8 .BULK.C009.C5	.	0.306	+INF	.
8 .BULK.C009.KC10	.	4.700	+INF	.
8 .BULK.C010.C141B	.	.	+INF	.
8 .BULK.C010.C5	.	0.539	+INF	.
8 .BULK.C010.KC10	.	.	+INF	EPS
8 .OVER.C007.C141B	.	.	+INF	0.247
8 .OVER.C007.C5	.	.	+INF	0.747
8 .OVER.C007.KC10	.	.	+INF	0.667
8 .OVER.C008.C141B	.	.	+INF	0.247
8 .OVER.C008.C5	.	.	+INF	0.747
8 .OVER.C008.KC10	.	.	+INF	0.667
8 .OVER.C009.C141B	.	.	+INF	0.247
8 .OVER.C009.C5	.	.	+INF	0.747
8 .OVER.C009.KC10	.	.	+INF	0.667
8 .OVER.C010.C141B	.	.	+INF	0.247
8 .OVER.C010.C5	.	.	+INF	0.747
8 .OVER.C010.KC10	.	.	+INF	0.667
8 .PAX .C007.C141B	.	.	+INF	0.247
8 .PAX .C007.C5	.	.	+INF	0.747
8 .PAX .C007.KC10	.	.	+INF	0.667
8 .PAX .C008.C141B	.	.	+INF	0.247
8 .PAX .C008.C5	.	.	+INF	0.747
8 .PAX .C008.KC10	.	.	+INF	0.667

8 .PAX .C009.C141B	.	.	+INF	0.247
8 .PAX .C009.C5	.	.	+INF	0.747
8 .PAX .C009.KC10	.	.	+INF	0.667
8 .PAX .C010.C141B	.	.	+INF	0.247
8 .PAX .C010.C5	.	.	+INF	0.747
8 .PAX .C010.KC10	.	.	+INF	0.667
9 .BULK.C008.C141B	.	.	+INF	.
9 .BULK.C008.C5	.	.	+INF	EPS
9 .BULK.C008.KC10	.	.	+INF	EPS
9 .BULK.C009.C141B	.	0.837	+INF	.
9 .BULK.C009.C5	.	0.155	+INF	.
9 .BULK.C009.KC10	.	.	+INF	.
9 .BULK.C010.C141B	.	0.163	+INF	.
9 .BULK.C010.C5	.	.	+INF	EPS
9 .BULK.C010.KC10	.	.	+INF	.
9 .BULK.C011.C141B	.	0.837	+INF	.
9 .BULK.C011.C5	.	0.461	+INF	.
9 .BULK.C011.KC10	.	4.700	+INF	.
9 .OVER.C008.C141B	.	.	+INF	0.247
9 .OVER.C008.C5	.	.	+INF	0.747
9 .OVER.C008.KC10	.	.	+INF	0.667
9 .OVER.C009.C141B	.	.	+INF	0.247
9 .OVER.C009.C5	.	.	+INF	0.747
9 .OVER.C009.KC10	.	.	+INF	0.667
9 .OVER.C010.C141B	.	.	+INF	0.247
9 .OVER.C010.C5	.	.	+INF	0.747
9 .OVER.C010.KC10	.	.	+INF	0.667
9 .OVER.C011.C141B	.	.	+INF	0.247
9 .OVER.C011.C5	.	.	+INF	0.747
9 .OVER.C011.KC10	.	.	+INF	0.667
9 .PAX .C008.C141B	.	.	+INF	0.247
9 .PAX .C008.C5	.	.	+INF	0.747
9 .PAX .C008.KC10	.	.	+INF	0.667
9 .PAX .C009.C141B	.	.	+INF	0.247
9 .PAX .C009.C5	.	.	+INF	0.747
9 .PAX .C009.KC10	.	.	+INF	0.667
9 .PAX .C010.C141B	.	.	+INF	0.247
9 .PAX .C010.C5	.	.	+INF	0.747
9 .PAX .C010.KC10	.	.	+INF	0.667
9 .PAX .C011.C141B	.	.	+INF	0.247
9 .PAX .C011.C5	.	.	+INF	0.747
9 .PAX .C011.KC10	.	.	+INF	0.667
10.BULK.C010.C141B	.	.	+INF	EPS
10.BULK.C010.C5	.	.	+INF	0.747
10.BULK.C010.KC10	.	.	+INF	EPS
10.BULK.C011.C141B	.	.	+INF	EPS

```
10.BULK.C011.C5             .         .         +INF        0.747
10.BULK.C011.KC10           .         .         +INF         EPS
10.BULK.C012.C141B          .         .         +INF         EPS
10.BULK.C012.C5             .         .         +INF         EPS
10.BULK.C012.KC10           .         .         +INF         EPS
10.OVER.C010.C141B          .         .         +INF          .
10.OVER.C010.C5             .         .         +INF        0.747
10.OVER.C010.KC10           .         .         +INF          .
10.OVER.C011.C141B          .         .         +INF          .
10.OVER.C011.C5             .         .         +INF        0.747
10.OVER.C011.KC10           .         .         +INF          .
10.OVER.C012.C141B          .         .         +INF          .
10.OVER.C012.C5             .       0.338       +INF          .
10.OVER.C012.KC10           .         .         +INF          .
10.PAX .C010.C141B          .         .         +INF         EPS
10.PAX .C010.C5             .         .         +INF        0.747
10.PAX .C010.KC10           .         .         +INF         EPS
10.PAX .C011.C141B          .         .         +INF         EPS
10.PAX .C011.C5             .         .         +INF        0.747
10.PAX .C011.KC10           .         .         +INF         EPS
10.PAX .C012.C141B          .         .         +INF         EPS
10.PAX .C012.C5             .         .         +INF         EPS
10.PAX .C012.KC10           .         .         +INF         EPS
---- VAR Y           number of type v lift assets to be acquired
         LOWER     LEVEL     UPPER    MARGINAL
C141B      .         .        +INF      0.259
C5         .         .        +INF      1.758
KC10       .       3.700      +INF        .
                           LOWER     LEVEL     UPPER    MARGINAL
---- VAR Z                 -INF      7.399     +INF
  Z          cost to be minimized
```

COST-MINIMIZATION RESULTS: INTEGER PROGRAM

```
set m movement requirements / 1 * 10 / ;

set v lift assets / c141b, c5,  kc10/ ;


set j / bulk, over, pax / ;

table capacity(v,j) capacity of vehicle v for cargo type j

          bulk    over    pax

```

```
  c141b    23.0     23.6      153
     c5    69.6     65.0      329
   kc10    62.1     26.4      257
   ;

table tons(m,j)

           bulk     over      pax

      1      15        0        0
      2      17        0        0
      3       0        0      125
      4       0       43       75
      5      71        0       55
      6      21        0       27
      7    37.5        0       25
      8     710        0        0
      9     377        0        0
     10       0       22        0
   ;

parameter L(m,j,v) load factors;
L(m,j,v) = tons(m,j)/capacity(v,j);

parameter a(m) availability of movement m
/
      1       1
      2       1
      3       1
      4       3
      5       4
      6       7
      7       6
      8       7
      9       8
     10      10
  /;

parameter lad(m) due date of movement m
/
      1       2
      2       2
      3       2
      4       5
```

```
        5       6
        6      10
        7       9
        8      11
        9      12
       10      13
   /;


set e list of unique poes
/
seattle
st-louis
boston
new-york
san-fran
san-dieg
 /;



set d list of unique pods
/
pingtung
chiayi
taipei
tainan
 /;

parameter poe(m,e) shows which poe shipment m uses

/
      1.seattle    = 1
      2.seattle    = 1
      3.st-louis    = 1
      4.st-louis    = 1
      5.st-louis    = 1
      6.boston    = 1
      7.new-york    = 1
      8.san-fran   = 1
      9.san-dieg    = 1
     10.san-fran    = 1
 / ;

parameter pod(m,d) shows which pod shipment m uses

/
```

```
      1.pingtung    = 1
      2.chiayi    = 1
      3.pingtung    = 1
      4.taipei    = 1
      5.taipei    = 1
      6.tainan    = 1
      7.tainan    = 1
      8.taipei    = 1
      9.pingtung    = 1
     10.pingtung    = 1
  / ;

parameter channel(e,d)

/
      seattle.pingtung = 1
      seattle.chiayi = 1
      st-louis.pingtung = 1
      st-louis.taipei = 1
      boston.tainan = 1
      new-york.tainan = 1
      san-fran.taipei = 1
      san-dieg.pingtung = 1
      san-fran.pingtung = 1
  / ;

parameter b(m,v) latest that m can go on v to meet lad ;
b(m,v) = lad(m) - 1;

set     t /c001   * c013 / ;

parameter tnum(t) number of day t;
        tnum(t) = ord(t);

set     h /c001   * c015 / ;

parameter hnum(h) number of day h;
    hnum(h) = ord(h);

parameter inuse(e,d,t)

/
      seattle.pingtung.c001 = 1
      seattle.chiayi.c001 = 1
      st-louis.pingtung.c001 = 1
      st-louis.taipei.c003 = 1
      st-louis.taipei.c004 = 1
```

```
      st-louis.taipei.c005 = 1
      boston.tainan.c007 = 1
      boston.tainan.c008 = 1
      boston.tainan.c009 = 1
      new-york.tainan.c006 = 1
      new-york.tainan.c007 = 1
      new-york.tainan.c008 = 1
      san-fran.taipei.c007 = 1
      san-fran.taipei.c008 = 1
      san-fran.taipei.c009 = 1
      san-fran.taipei.c010 = 1
      san-dieg.pingtung.c008 = 1
      san-dieg.pingtung.c009 = 1
      san-dieg.pingtung.c010 = 1
      san-dieg.pingtung.c011 = 1
      san-fran.pingtung.c010 = 1
      san-fran.pingtung.c011 = 1
      san-fran.pingtung.c012 = 1
  / ;

parameter dollars(v) guesstimate cost of additional lift assets of type v
        /
        c141b    1
        c5       4
        kc10     2
        / ;

parameter S(e,d,v) cycle time;
S(e,d,v) = 1;

parameter N(v) number of lift assets of type v at on hand
        /
        c141b    1
        c5       1
        kc10     1
        / ;

variables
       U(e, d, t, v) no. of v loaded at POE e in t bound for POD d
       X(m, j, t, v) no. of v loaded with j of m on day t
       Y(v) number of type v lift assets to be acquired
       z cost to be minimized
;

positive variables X;
integer variables U, Y;

```

```
equations
    cost define objective function
    cost2 dummy constraint
    demand(m,j) every movement is shipped in its entirety
    supply1(e,d,t,v) supply of shipping
    supply2(h,v) supply of shipping;
cost .. z =e= sum(v, dollars(v)*Y(v)) ;
demand(m,j) $ (tons(m,j)).. sum( (v,t) $
(tnum(t) ge A(m) and tnum(t) le B(m,v)),
  (1/L(m,j,v))*X(m,j,t,v) ) =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
.. sum( (m,j)$ (poe(m,e) and pod(m,d) and
(tnum(t) ge A(m) and tnum(t) le B(m,v))), X(m,j,t,v) )- U(e,d,t,v) =l=  0 ;
supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
and inuse(e,d,t)), U(e,d,t,v) )
                - Y(v) =l= N(v) ;
model smmIP / demand, supply1, supply2, cost /;
option mip=lamps, iterlim=10000;
solve smmIP using mip minimizing z ;
```

SETS

D	list of unique pods
E	list of unique poes
H	
J	
M	movement requirements
T	
V	lift assets

PARAMETERS

A	availability of movement m
B	latest that m can go on v to meet lad
CAPACITY	capacity of vehicle v for cargo type j
CHANNEL	
DOLLARS	guesstimate cost of additional lift assets of type v
HNUM	number of day h
INUSE	

```
L             load factors
LAD           due date of movement m
N             number of lift assets of type v at on hand
POD           shows which pod shipment m uses
POE           shows which poe shipment m uses
S             cycle time
TNUM          number of day t
TONS
VARIABLES
U             no. of v loaded at POE e in t bound for POD d
X             no. of v loaded with j of m on day t
Y             number of type v lift assets to be acquired
Z             cost to be minimized
EQUATIONS
COST          define objective function
COST2         dummy constraint
DEMAND        every movement is shipped in its entirety
SUPPLY1       supply of shipping
SUPPLY2       supply of shipping
---- DEMAND      =E=  every movement is shipped in its entirety
DEMAND(1,BULK)..  1.5333*X(1,BULK,C001,C141B) + 4.64*X(1,BULK,C001,C5)
      + 4.14*X(1,BULK,C001,KC10) =E= 1 ; (LHS = 0 ***)
DEMAND(2,BULK)..  1.3529*X(2,BULK,C001,C141B) + 4.0941*X(2,BULK,C001,C5)
      + 3.6529*X(2,BULK,C001,KC10) =E= 1 ; (LHS = 0 ***)
DEMAND(3,PAX)..  1.224*X(3,PAX,C001,C141B) + 2.632*X(3,PAX,C001,C5)
      + 2.056*X(3,PAX,C001,KC10) =E= 1 ; (LHS = 0 ***)
REMAINING 11 ENTRIES SKIPPED
---- SUPPLY1     =L=  supply of shipping
SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)..  - U(SEATTLE,PINGTUNG,C001,C141B)
      + X(1,BULK,C001,C141B) + X(1,OVER,C001,C141B) + X(1,PAX,C001,C141B) =L= 0 ;
      (LHS = 0)
SUPPLY1(SEATTLE,PINGTUNG,C001,C5)..  - U(SEATTLE,PINGTUNG,C001,C5)
      + X(1,BULK,C001,C5) + X(1,OVER,C001,C5) + X(1,PAX,C001,C5) =L= 0 ;
      (LHS = 0)
SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)..  - U(SEATTLE,PINGTUNG,C001,KC10)
      + X(1,BULK,C001,KC10) + X(1,OVER,C001,KC10) + X(1,PAX,C001,KC10) =L= 0 ;
      (LHS = 0)
REMAINING 66 ENTRIES SKIPPED
---- SUPPLY2     =L=  supply of shipping
SUPPLY2(C001,C141B)..  U(SEATTLE,PINGTUNG,C001,C141B)
      + U(SEATTLE,CHIAYI,C001,C141B) + U(ST-LOUIS,PINGTUNG,C001,C141B)
      - Y(C141B) =L= 1 ; (LHS = 0)
SUPPLY2(C001,C5)..  U(SEATTLE,PINGTUNG,C001,C5) + U(SEATTLE,CHIAYI,C001,C5)
      + U(ST-LOUIS,PINGTUNG,C001,C5) - Y(C5) =L= 1 ; (LHS = 0)
SUPPLY2(C001,KC10)..  U(SEATTLE,PINGTUNG,C001,KC10)
      + U(SEATTLE,CHIAYI,C001,KC10) + U(ST-LOUIS,PINGTUNG,C001,KC10)
      - Y(KC10) =L= 1 ; (LHS = 0)
```

```
REMAINING 42 ENTRIES SKIPPED
---- COST        =E=  define objective function
COST..  - Y(C141B) - 4*Y(C5) - 2*Y(KC10) + Z =E= 0 ; (LHS = 0)
---- U           no. of v loaded at POE e in t bound for POD d
U(SEATTLE,PINGTUNG,C001,C141B)
                 (.LO, .L, .UP = 0, 0, 100)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)
        1        SUPPLY2(C001,C141B)
        1        SUPPLY2(C002,C141B)
U(SEATTLE,PINGTUNG,C001,C5)
                 (.LO, .L, .UP = 0, 0, 100)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,C5)
        1        SUPPLY2(C001,C5)
        1        SUPPLY2(C002,C5)
U(SEATTLE,PINGTUNG,C001,KC10)
                 (.LO, .L, .UP = 0, 0, 100)
       -1        SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)
        1        SUPPLY2(C001,KC10)
        1        SUPPLY2(C002,KC10)
REMAINING 66 ENTRIES SKIPPED
---- X           no. of v loaded with j of m on day t
X(1,BULK,C001,C141B)
                 (.LO, .L, .UP = 0, 0, +INF)
        1.5333   DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,C141B)
X(1,BULK,C001,C5)
                 (.LO, .L, .UP = 0, 0, +INF)
        4.64     DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,C5)
X(1,BULK,C001,KC10)
                 (.LO, .L, .UP = 0, 0, +INF)
        4.14     DEMAND(1,BULK)
        1        SUPPLY1(SEATTLE,PINGTUNG,C001,KC10)
REMAINING 213 ENTRIES SKIPPED
---- Y           number of type v lift assets to be acquired
Y(C141B)
                 (.LO, .L, .UP = 0, 0, 100)
       -1        SUPPLY2(C001,C141B)
       -1        SUPPLY2(C002,C141B)
       -1        SUPPLY2(C003,C141B)
       -1        SUPPLY2(C004,C141B)
       -1        SUPPLY2(C005,C141B)
       -1        SUPPLY2(C006,C141B)
       -1        SUPPLY2(C007,C141B)
       -1        SUPPLY2(C008,C141B)
       -1        SUPPLY2(C009,C141B)
       -1        SUPPLY2(C010,C141B)
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        -1      SUPPLY2(C011,C141B)
        -1      SUPPLY2(C012,C141B)
        -1      SUPPLY2(C013,C141B)
        -1      SUPPLY2(C014,C141B)
        -1      SUPPLY2(C015,C141B)
        -1      COST
Y(C5)
                (.LO, .L, .UP = 0, 0, 100)
        -1      SUPPLY2(C001,C5)
        -1      SUPPLY2(C002,C5)
        -1      SUPPLY2(C003,C5)
        -1      SUPPLY2(C004,C5)
        -1      SUPPLY2(C005,C5)
        -1      SUPPLY2(C006,C5)
        -1      SUPPLY2(C007,C5)
        -1      SUPPLY2(C008,C5)
        -1      SUPPLY2(C009,C5)
        -1      SUPPLY2(C010,C5)
        -1      SUPPLY2(C011,C5)
        -1      SUPPLY2(C012,C5)
        -1      SUPPLY2(C013,C5)
        -1      SUPPLY2(C014,C5)
        -1      SUPPLY2(C015,C5)
        -4      COST
Y(KC10)
                (.LO, .L, .UP = 0, 0, 100)
        -1      SUPPLY2(C001,KC10)
        -1      SUPPLY2(C002,KC10)
        -1      SUPPLY2(C003,KC10)
        -1      SUPPLY2(C004,KC10)
        -1      SUPPLY2(C005,KC10)
        -1      SUPPLY2(C006,KC10)
        -1      SUPPLY2(C007,KC10)
        -1      SUPPLY2(C008,KC10)
        -1      SUPPLY2(C009,KC10)
        -1      SUPPLY2(C010,KC10)
        -1      SUPPLY2(C011,KC10)
        -1      SUPPLY2(C012,KC10)
        -1      SUPPLY2(C013,KC10)
        -1      SUPPLY2(C014,KC10)
        -1      SUPPLY2(C015,KC10)
        -2      COST
---- Z          cost to be minimized
Z
                (.LO, .L, .UP = -INF, 0, +INF)
         1      COST
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---- EQU DEMAND        every movement is shipped in its entirety
               LOWER     LEVEL     UPPER    MARGINAL
1 .BULK        1.000     1.000     1.000      EPS
2 .BULK        1.000     1.000     1.000      EPS
3 .PAX         1.000     1.000     1.000      EPS
4 .OVER        1.000     1.000     1.000      EPS
4 .PAX         1.000     1.000     1.000      EPS
5 .BULK        1.000     1.000     1.000      EPS
5 .PAX         1.000     1.000     1.000      EPS
6 .BULK        1.000     1.000     1.000      EPS
6 .PAX         1.000     1.000     1.000      EPS
7 .BULK        1.000     1.000     1.000      EPS
7 .PAX         1.000     1.000     1.000      EPS
8 .BULK        1.000     1.000     1.000      EPS
9 .BULK        1.000     1.000     1.000      EPS
10.OVER        1.000     1.000     1.000      EPS
---- EQU SUPPLY1       supply of shipping
                                    LOWER     LEVEL     UPPER    MARGINAL
SEATTLE .PINGTUNG.C001.C141B        -INF        .         .        EPS
SEATTLE .PINGTUNG.C001.C5           -INF        .         .         .
SEATTLE .PINGTUNG.C001.KC10         -INF        .         .        EPS
SEATTLE .CHIAYI  .C001.C141B        -INF        .         .        EPS
SEATTLE .CHIAYI  .C001.C5           -INF        .         .        EPS
SEATTLE .CHIAYI  .C001.KC10         -INF        .         .        EPS
ST-LOUIS.PINGTUNG.C001.C141B        -INF        .         .        EPS
ST-LOUIS.PINGTUNG.C001.C5           -INF        .         .        EPS
ST-LOUIS.PINGTUNG.C001.KC10         -INF        .         .         .
ST-LOUIS.TAIPEI  .C003.C141B        -INF        .         .         .
ST-LOUIS.TAIPEI  .C003.C5           -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C003.KC10         -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C004.C141B        -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C004.C5           -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C004.KC10         -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C005.C141B        -INF        .         .         .
ST-LOUIS.TAIPEI  .C005.C5           -INF        .         .        EPS
ST-LOUIS.TAIPEI  .C005.KC10         -INF        .         .         .
BOSTON  .TAINAN  .C007.C141B        -INF        .         .        EPS
BOSTON  .TAINAN  .C007.C5           -INF        .         .        EPS
BOSTON  .TAINAN  .C007.KC10         -INF        .         .        EPS
BOSTON  .TAINAN  .C008.C141B        -INF        .         .        EPS
BOSTON  .TAINAN  .C008.C5           -INF        .         .        EPS
BOSTON  .TAINAN  .C008.KC10         -INF        .         .        EPS
BOSTON  .TAINAN  .C009.C141B        -INF        .         .        EPS
BOSTON  .TAINAN  .C009.C5           -INF        .         .        EPS
BOSTON  .TAINAN  .C009.KC10         -INF        .         .        EPS
NEW-YORK.TAINAN  .C006.C141B        -INF        .         .        EPS
NEW-YORK.TAINAN  .C006.C5           -INF        .         .        EPS
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NEW-YORK.TAINAN  .C006.KC10     -INF     .      .      EPS
NEW-YORK.TAINAN  .C007.C141B    -INF     .      .      EPS
NEW-YORK.TAINAN  .C007.C5       -INF     .      .      EPS
NEW-YORK.TAINAN  .C007.KC10     -INF     .      .      EPS
NEW-YORK.TAINAN  .C008.C141B    -INF     .      .      EPS
NEW-YORK.TAINAN  .C008.C5       -INF     .      .      EPS
NEW-YORK.TAINAN  .C008.KC10     -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C010.C141B    -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C010.C5       -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C010.KC10     -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C011.C141B    -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C011.C5       -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C011.KC10     -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C012.C141B    -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C012.C5       -INF     .      .      EPS
SAN-FRAN.PINGTUNG.C012.KC10     -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C007.C141B    -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C007.C5       -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C007.KC10     -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C008.C141B    -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C008.C5       -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C008.KC10     -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C009.C141B    -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C009.C5       -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C009.KC10     -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C010.C141B    -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C010.C5       -INF     .      .      EPS
SAN-FRAN.TAIPEI  .C010.KC10     -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C008.C141B    -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C008.C5       -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C008.KC10     -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C009.C141B    -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C009.C5       -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C009.KC10     -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C010.C141B    -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C010.C5       -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C010.KC10     -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C011.C141B    -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C011.C5       -INF     .      .      EPS
SAN-DIEG.PINGTUNG.C011.KC10     -INF     .      .      EPS
---- EQU SUPPLY2      supply of shipping
              LOWER     LEVEL     UPPER    MARGINAL
C001.C141B     -INF       .       1.000       .
C001.C5        -INF     1.000     1.000       .
C001.KC10      -INF    -2.000     1.000       .
C002.C141B     -INF       .       1.000       .
C002.C5        -INF     1.000     1.000      EPS
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C002.KC10        -INF     -2.000     1.000      .
C003.C141B       -INF       .        1.000      .
C003.C5          -INF      1.000     1.000      .
C003.KC10        -INF     -4.000     1.000      .
C004.C141B       -INF       .        1.000      .
C004.C5          -INF      1.000     1.000     EPS
C004.KC10        -INF     -3.000     1.000      .
C005.C141B       -INF       .        1.000      .
C005.C5          -INF      1.000     1.000     EPS
C005.KC10        -INF     -3.000     1.000      .
C006.C141B       -INF      1.000     1.000      .
C006.C5          -INF      1.000     1.000     EPS
C006.KC10        -INF     -4.000     1.000      .
C007.C141B       -INF      1.000     1.000     EPS
C007.C5          -INF      1.000     1.000     EPS
C007.KC10        -INF      1.000     1.000     EPS
C008.C141B       -INF      1.000     1.000     EPS
C008.C5          -INF      1.000     1.000     EPS
C008.KC10        -INF      1.000     1.000     EPS
C009.C141B       -INF      1.000     1.000     EPS
C009.C5          -INF      1.000     1.000     EPS
C009.KC10        -INF      1.000     1.000     EPS
C010.C141B       -INF      1.000     1.000     EPS
C010.C5          -INF      1.000     1.000     EPS
C010.KC10        -INF      1.000     1.000      .
C011.C141B       -INF      1.000     1.000     EPS
C011.C5          -INF      1.000     1.000      .
C011.KC10        -INF      1.000     1.000     EPS
C012.C141B       -INF      1.000     1.000      .
C012.C5          -INF      1.000     1.000     EPS
C012.KC10        -INF      1.000     1.000     EPS
C013.C141B       -INF      1.000     1.000      .
C013.C5          -INF       .        1.000      .
C013.KC10        -INF     -4.000     1.000      .
C014.C141B       -INF       .        1.000      .
C014.C5          -INF       .        1.000      .
C014.KC10        -INF     -4.000     1.000      .
C015.C141B       -INF       .        1.000      .
C015.C5          -INF       .        1.000      .
C015.KC10        -INF     -4.000     1.000      .
Solution Report     SOLVE SMMIP USING MIP FROM LINE 226
                          LOWER     LEVEL     UPPER     MARGINAL
---- EQU COST               .         .         .         1.000
  COST         define objective function
---- VAR U             no. of v loaded at POE e in t bound for POD d
                                    LOWER     LEVEL     UPPER     MARGINAL
SEATTLE .PINGTUNG.C001.C141B          .         .      100.000      EPS
SEATTLE .PINGTUNG.C001.C5             .         .      100.000      EPS
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SEATTLE .PINGTUNG.C001.KC10       .     1.000   100.000      EPS
SEATTLE .CHIAYI  .C001.C141B      .       .     100.000      EPS
SEATTLE .CHIAYI  .C001.C5         .       .     100.000       .
SEATTLE .CHIAYI  .C001.KC10       .     1.000   100.000      EPS
ST-LOUIS.PINGTUNG.C001.C141B      .       .     100.000      EPS
ST-LOUIS.PINGTUNG.C001.C5         .     1.000   100.000      EPS
ST-LOUIS.PINGTUNG.C001.KC10       .       .     100.000      EPS
ST-LOUIS.TAIPEI  .C003.C141B      .       .     100.000      EPS
ST-LOUIS.TAIPEI  .C003.C5         .     1.000   100.000       .
ST-LOUIS.TAIPEI  .C003.KC10       .       .     100.000      EPS
ST-LOUIS.TAIPEI  .C004.C141B      .       .     100.000      EPS
ST-LOUIS.TAIPEI  .C004.C5         .       .     100.000       .
ST-LOUIS.TAIPEI  .C004.KC10       .     1.000   100.000      EPS
ST-LOUIS.TAIPEI  .C005.C141B      .       .     100.000      EPS
ST-LOUIS.TAIPEI  .C005.C5         .     1.000   100.000       .
ST-LOUIS.TAIPEI  .C005.KC10       .       .     100.000      EPS
BOSTON  .TAINAN  .C007.C141B      .       .     100.000      EPS
BOSTON  .TAINAN  .C007.C5         .     1.000   100.000      EPS
BOSTON  .TAINAN  .C007.KC10       .       .     100.000      EPS
BOSTON  .TAINAN  .C008.C141B      .       .     100.000      EPS
BOSTON  .TAINAN  .C008.C5         .       .     100.000      EPS
BOSTON  .TAINAN  .C008.KC10       .       .     100.000      EPS
BOSTON  .TAINAN  .C009.C141B      .       .     100.000       .
BOSTON  .TAINAN  .C009.C5         .       .     100.000      EPS
BOSTON  .TAINAN  .C009.KC10       .       .     100.000      EPS
NEW-YORK.TAINAN  .C006.C141B      .     1.000   100.000       .
NEW-YORK.TAINAN  .C006.C5         .       .     100.000       .
NEW-YORK.TAINAN  .C006.KC10       .       .     100.000       .
NEW-YORK.TAINAN  .C007.C141B      .       .     100.000       .
NEW-YORK.TAINAN  .C007.C5         .       .     100.000       .
NEW-YORK.TAINAN  .C007.KC10       .       .     100.000       .
NEW-YORK.TAINAN  .C008.C141B      .     1.000   100.000      EPS
NEW-YORK.TAINAN  .C008.C5         .       .     100.000       .
NEW-YORK.TAINAN  .C008.KC10       .       .     100.000       .
SAN-FRAN.PINGTUNG.C010.C141B      .       .     100.000      EPS
SAN-FRAN.PINGTUNG.C010.C5         .       .     100.000       .
SAN-FRAN.PINGTUNG.C010.KC10       .       .     100.000      EPS
SAN-FRAN.PINGTUNG.C011.C141B      .       .     100.000       .
SAN-FRAN.PINGTUNG.C011.C5         .       .     100.000      EPS
SAN-FRAN.PINGTUNG.C011.KC10       .       .     100.000      EPS
SAN-FRAN.PINGTUNG.C012.C141B      .     1.000   100.000      EPS
SAN-FRAN.PINGTUNG.C012.C5         .       .     100.000       .
SAN-FRAN.PINGTUNG.C012.KC10       .       .     100.000       .
SAN-FRAN.TAIPEI  .C007.C141B      .       .     100.000      EPS
SAN-FRAN.TAIPEI  .C007.C5         .       .     100.000      EPS
SAN-FRAN.TAIPEI  .C007.KC10       .     5.000   100.000      EPS
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SAN-FRAN.TAIPEI   .C008.C141B      .        .      100.000      .
SAN-FRAN.TAIPEI   .C008.C5         .        .      100.000     EPS
SAN-FRAN.TAIPEI   .C008.KC10       .        .      100.000     EPS
SAN-FRAN.TAIPEI   .C009.C141B      .        .      100.000     EPS
SAN-FRAN.TAIPEI   .C009.C5         .      1.000    100.000     EPS
SAN-FRAN.TAIPEI   .C009.KC10       .      5.000    100.000     EPS
SAN-FRAN.TAIPEI   .C010.C141B      .      1.000    100.000      .
SAN-FRAN.TAIPEI   .C010.C5         .        .      100.000     EPS
SAN-FRAN.TAIPEI   .C010.KC10       .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C008.C141B       .        .      100.000      .
SAN-DIEG.PINGTUNG.C008.C5          .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C008.KC10        .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C009.C141B       .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C009.C5          .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C009.KC10        .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C010.C141B       .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C010.C5          .        .      100.000     EPS
SAN-DIEG.PINGTUNG.C010.KC10        .        .      100.000      .
SAN-DIEG.PINGTUNG.C011.C141B       .        .      100.000      .
SAN-DIEG.PINGTUNG.C011.C5          .      1.000    100.000     EPS
SAN-DIEG.PINGTUNG.C011.KC10        .      5.000    100.000     EPS
---- VAR X              no. of v loaded with j of m on day t
                          LOWER     LEVEL     UPPER    MARGINAL
1 .BULK.C001.C141B          .         .       +INF        .
1 .BULK.C001.C5             .         .       +INF       EPS
1 .BULK.C001.KC10           .       0.242     +INF        .
1 .OVER.C001.C141B          .         .       +INF       EPS
1 .OVER.C001.C5             .         .       +INF       EPS
1 .OVER.C001.KC10           .         .       +INF       EPS
1 .PAX .C001.C141B          .         .       +INF       EPS
1 .PAX .C001.C5             .         .       +INF       EPS
1 .PAX .C001.KC10           .       0.758     +INF        .
2 .BULK.C001.C141B          .         .       +INF        .
2 .BULK.C001.C5             .         .       +INF       EPS
2 .BULK.C001.KC10           .       0.274     +INF        .
2 .OVER.C001.C141B          .         .       +INF       EPS
2 .OVER.C001.C5             .         .       +INF       EPS
2 .OVER.C001.KC10           .         .       +INF       EPS
2 .PAX .C001.C141B          .         .       +INF       EPS
2 .PAX .C001.C5             .         .       +INF        .
2 .PAX .C001.KC10           .       0.726     +INF        .
3 .BULK.C001.C141B          .         .       +INF       EPS
3 .BULK.C001.C5             .         .       +INF       EPS
3 .BULK.C001.KC10           .         .       +INF       EPS
3 .OVER.C001.C141B          .         .       +INF       EPS
3 .OVER.C001.C5             .       0.620     +INF        .
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3 .OVER.C001.KC10      .        .     +INF    EPS
3 .PAX .C001.C141B     .        .     +INF     .
3 .PAX .C001.C5        .      0.380   +INF     .
3 .PAX .C001.KC10      .        .     +INF    EPS
4 .BULK.C003.C141B     .        .     +INF    EPS
4 .BULK.C003.C5        .      0.110   +INF     .
4 .BULK.C003.KC10      .        .     +INF    EPS
4 .BULK.C004.C141B     .        .     +INF    EPS
4 .BULK.C004.C5        .        .     +INF    EPS
4 .BULK.C004.KC10      .        .     +INF    EPS
4 .OVER.C003.C141B     .        .     +INF    EPS
4 .OVER.C003.C5        .      0.662   +INF     .
4 .OVER.C003.KC10      .        .     +INF    EPS
4 .OVER.C004.C141B     .        .     +INF    EPS
4 .OVER.C004.C5        .        .     +INF    EPS
4 .OVER.C004.KC10      .        .     +INF    EPS
4 .PAX .C003.C141B     .        .     +INF    EPS
4 .PAX .C003.C5        .      0.228   +INF     .
4 .PAX .C003.KC10      .        .     +INF     .
4 .PAX .C004.C141B     .        .     +INF    EPS
4 .PAX .C004.C5        .        .     +INF    EPS
4 .PAX .C004.KC10      .        .     +INF    EPS
5 .BULK.C004.C141B     .        .     +INF     .
5 .BULK.C004.C5        .        .     +INF     .
5 .BULK.C004.KC10      .      0.786   +INF     .
5 .BULK.C005.C141B     .        .     +INF    EPS
5 .BULK.C005.C5        .      0.319   +INF     .
5 .BULK.C005.KC10      .        .     +INF    EPS
5 .OVER.C004.C141B     .        .     +INF    EPS
5 .OVER.C004.C5        .        .     +INF    EPS
5 .OVER.C004.KC10      .        .     +INF    EPS
5 .OVER.C005.C141B     .        .     +INF    EPS
5 .OVER.C005.C5        .      0.681   +INF     .
5 .OVER.C005.KC10      .        .     +INF    EPS
5 .PAX .C004.C141B     .        .     +INF    EPS
5 .PAX .C004.C5        .        .     +INF    EPS
5 .PAX .C004.KC10      .      0.214   +INF     .
5 .PAX .C005.C141B     .        .     +INF    EPS
5 .PAX .C005.C5        .        .     +INF    EPS
5 .PAX .C005.KC10      .        .     +INF    EPS
6 .BULK.C007.C141B     .        .     +INF    EPS
6 .BULK.C007.C5        .      0.302   +INF     .
6 .BULK.C007.KC10      .        .     +INF     .
6 .BULK.C008.C141B     .        .     +INF     .
6 .BULK.C008.C5        .        .     +INF    EPS
6 .BULK.C008.KC10      .        .     +INF     .
```

6 .BULK.C009.C141B	.	.	+INF	.
6 .BULK.C009.C5	.	.	+INF	EPS
6 .BULK.C009.KC10	.	.	+INF	.
6 .OVER.C007.C141B	.	.	+INF	EPS
6 .OVER.C007.C5	.	0.616	+INF	.
6 .OVER.C007.KC10	.	.	+INF	EPS
6 .OVER.C008.C141B	.	.	+INF	EPS
6 .OVER.C008.C5	.	.	+INF	.
6 .OVER.C008.KC10	.	.	+INF	EPS
6 .OVER.C009.C141B	.	.	+INF	EPS
6 .OVER.C009.C5	.	.	+INF	.
6 .OVER.C009.KC10	.	.	+INF	EPS
6 .PAX .C007.C141B	.	.	+INF	.
6 .PAX .C007.C5	.	0.082	+INF	.
6 .PAX .C007.KC10	.	.	+INF	EPS
6 .PAX .C008.C141B	.	.	+INF	EPS
6 .PAX .C008.C5	.	.	+INF	EPS
6 .PAX .C008.KC10	.	.	+INF	EPS
6 .PAX .C009.C141B	.	.	+INF	EPS
6 .PAX .C009.C5	.	.	+INF	EPS
6 .PAX .C009.KC10	.	.	+INF	EPS
7 .BULK.C006.C141B	.	1.000	+INF	.
7 .BULK.C006.C5	.	.	+INF	EPS
7 .BULK.C006.KC10	.	.	+INF	EPS
7 .BULK.C007.C141B	.	.	+INF	EPS
7 .BULK.C007.C5	.	.	+INF	EPS
7 .BULK.C007.KC10	.	.	+INF	EPS
7 .BULK.C008.C141B	.	0.630	+INF	.
7 .BULK.C008.C5	.	.	+INF	EPS
7 .BULK.C008.KC10	.	.	+INF	EPS
7 .OVER.C006.C141B	.	.	+INF	EPS
7 .OVER.C006.C5	.	.	+INF	.
7 .OVER.C006.KC10	.	.	+INF	.
7 .OVER.C007.C141B	.	.	+INF	EPS
7 .OVER.C007.C5	.	.	+INF	.
7 .OVER.C007.KC10	.	.	+INF	.
7 .OVER.C008.C141B	.	0.206	+INF	.
7 .OVER.C008.C5	.	.	+INF	.
7 .OVER.C008.KC10	.	.	+INF	.
7 .PAX .C006.C141B	.	.	+INF	EPS
7 .PAX .C006.C5	.	.	+INF	EPS
7 .PAX .C006.KC10	.	.	+INF	EPS
7 .PAX .C007.C141B	.	.	+INF	EPS
7 .PAX .C007.C5	.	.	+INF	EPS
7 .PAX .C007.KC10	.	.	+INF	EPS
7 .PAX .C008.C141B	.	0.163	+INF	.

7 .PAX .C008.C5	.	.	+INF	EPS
7 .PAX .C008.KC10	.	.	+INF	EPS
8 .BULK.C007.C141B	.	.	+INF	.
8 .BULK.C007.C5	.	.	+INF	.
8 .BULK.C007.KC10	.	5.000	+INF	.
8 .BULK.C008.C141B	.	.	+INF	.
8 .BULK.C008.C5	.	.	+INF	.
8 .BULK.C008.KC10	.	.	+INF	.
8 .BULK.C009.C141B	.	.	+INF	.
8 .BULK.C009.C5	.	0.948	+INF	.
8 .BULK.C009.KC10	.	5.000	+INF	.
8 .BULK.C010.C141B	.	1.000	+INF	.
8 .BULK.C010.C5	.	.	+INF	EPS
8 .BULK.C010.KC10	.	.	+INF	.
8 .OVER.C007.C141B	.	.	+INF	EPS
8 .OVER.C007.C5	.	.	+INF	EPS
8 .OVER.C007.KC10	.	.	+INF	EPS
8 .OVER.C008.C141B	.	.	+INF	EPS
8 .OVER.C008.C5	.	.	+INF	EPS
8 .OVER.C008.KC10	.	.	+INF	EPS
8 .OVER.C009.C141B	.	.	+INF	EPS
8 .OVER.C009.C5	.	.	+INF	EPS
8 .OVER.C009.KC10	.	.	+INF	EPS
8 .OVER.C010.C141B	.	.	+INF	EPS
8 .OVER.C010.C5	.	.	+INF	EPS
8 .OVER.C010.KC10	.	.	+INF	EPS
8 .PAX .C007.C141B	.	.	+INF	EPS
8 .PAX .C007.C5	.	.	+INF	EPS
8 .PAX .C007.KC10	.	.	+INF	EPS
8 .PAX .C008.C141B	.	.	+INF	EPS
8 .PAX .C008.C5	.	.	+INF	EPS
8 .PAX .C008.KC10	.	.	+INF	EPS
8 .PAX .C009.C141B	.	.	+INF	EPS
8 .PAX .C009.C5	.	0.052	+INF	.
8 .PAX .C009.KC10	.	.	+INF	EPS
8 .PAX .C010.C141B	.	.	+INF	EPS
8 .PAX .C010.C5	.	.	+INF	.
8 .PAX .C010.KC10	.	.	+INF	EPS
9 .BULK.C008.C141B	.	.	+INF	EPS
9 .BULK.C008.C5	.	.	+INF	.
9 .BULK.C008.KC10	.	.	+INF	.
9 .BULK.C009.C141B	.	.	+INF	.
9 .BULK.C009.C5	.	.	+INF	.
9 .BULK.C009.KC10	.	.	+INF	.
9 .BULK.C010.C141B	.	.	+INF	.
9 .BULK.C010.C5	.	.	+INF	.

9 .BULK.C010.KC10	.	.	+INF	.
9 .BULK.C011.C141B	.	.	+INF	EPS
9 .BULK.C011.C5	.	0.955	+INF	.
9 .BULK.C011.KC10	.	5.000	+INF	.
9 .OVER.C008.C141B	.	.	+INF	EPS
9 .OVER.C008.C5	.	.	+INF	EPS
9 .OVER.C008.KC10	.	.	+INF	EPS
9 .OVER.C009.C141B	.	.	+INF	EPS
9 .OVER.C009.C5	.	.	+INF	EPS
9 .OVER.C009.KC10	.	.	+INF	EPS
9 .OVER.C010.C141B	.	.	+INF	EPS
9 .OVER.C010.C5	.	.	+INF	EPS
9 .OVER.C010.KC10	.	.	+INF	EPS
9 .OVER.C011.C141B	.	.	+INF	EPS
9 .OVER.C011.C5	.	.	+INF	EPS
9 .OVER.C011.KC10	.	.	+INF	EPS
9 .PAX .C008.C141B	.	.	+INF	EPS
9 .PAX .C008.C5	.	.	+INF	EPS
9 .PAX .C008.KC10	.	.	+INF	EPS
9 .PAX .C009.C141B	.	.	+INF	EPS
9 .PAX .C009.C5	.	.	+INF	EPS
9 .PAX .C009.KC10	.	.	+INF	EPS
9 .PAX .C010.C141B	.	.	+INF	EPS
9 .PAX .C010.C5	.	.	+INF	EPS
9 .PAX .C010.KC10	.	.	+INF	EPS
9 .PAX .C011.C141B	.	.	+INF	EPS
9 .PAX .C011.C5	.	0.045	+INF	.
9 .PAX .C011.KC10	.	.	+INF	EPS
10.BULK.C010.C141B	.	.	+INF	EPS
10.BULK.C010.C5	.	.	+INF	EPS
10.BULK.C010.KC10	.	.	+INF	EPS
10.BULK.C011.C141B	.	.	+INF	EPS
10.BULK.C011.C5	.	.	+INF	EPS
10.BULK.C011.KC10	.	.	+INF	EPS
10.BULK.C012.C141B	.	.	+INF	EPS
10.BULK.C012.C5	.	.	+INF	EPS
10.BULK.C012.KC10	.	.	+INF	EPS
10.OVER.C010.C141B	.	.	+INF	.
10.OVER.C010.C5	.	.	+INF	EPS
10.OVER.C010.KC10	.	.	+INF	EPS
10.OVER.C011.C141B	.	.	+INF	.
10.OVER.C011.C5	.	.	+INF	EPS
10.OVER.C011.KC10	.	.	+INF	EPS
10.OVER.C012.C141B	.	0.932	+INF	.
10.OVER.C012.C5	.	.	+INF	EPS
10.OVER.C012.KC10	.	.	+INF	EPS

```
10.PAX .C010.C141B        .          .         +INF       EPS
10.PAX .C010.C5           .          .         +INF        .
10.PAX .C010.KC10         .          .         +INF        .
10.PAX .C011.C141B        .          .         +INF       EPS
10.PAX .C011.C5           .          .         +INF        .
10.PAX .C011.KC10         .          .         +INF        .
10.PAX .C012.C141B        .        0.068       +INF        .
10.PAX .C012.C5           .          .         +INF        .
10.PAX .C012.KC10         .          .         +INF        .
---- VAR Y             number of type v lift assets to be acquired
         LOWER      LEVEL      UPPER     MARGINAL
C141B      .          .       100.000     1.000
C5         .          .       100.000     4.000
KC10       .        4.000     100.000     2.000
                         LOWER      LEVEL      UPPER     MARGINAL
---- VAR Z               -INF       8.000      +INF         .
  Z            cost to be minimized
```

SAMPLE PROGRAMS FOR REALISTIC DATA AND ANALYSIS

The next programs use the data described in Chapter Five and show exactly how the analysis was done on a relatively large data set. The first listing shows the data structures shared by all the variants of the model, and the rest of the listings show the GAMS version of the model equations for each of the models. A complete, working program can be formed by concatenating the data structures program with any of the equation model programs. The programs listed show the "min-cost", "min-lateness", "min-earlyness", and "min-prepo" models actually used to produce the results listed in Chapter Five.

GAMS Program Data Structures

```
set m movement requirements / 1 * 51 / ;
set v lift assets / c5, c141b, c17, lrwc, lrwp, bulk, cont, roro / ;
set j / bulk, over, out, pax, avn, whl, track, cont, other / ;
table capacity(v,j) capacity of vehicle v for cargo type j
        bulk    over     out     pax     avn     whl   track    cont   other
   c5   75.6    75.6    75.6       0       0       0       0    75.6    75.6
c141b   26.0    26.0       0       0       0       0       0       0    26.0
  c17   40.0    40.0    40.0       0       0       0       0    40.0    40.0
 lrwc   69.6    65.0    66.4       0       0       0       0    66.4    66.4
 lrwp      0       0       0     329       0       0       0       0       0
 bulk  15000   15000   15000       0   15000       0       0       0   15000
 cont  38000   38000   38000       0       0       0       0   38000       0
 roro  38000   38000   38000       0       0   38000   38000       0       0
;
```

table tons(m,j)

	bulk	over	out	pax	avn	whl	track	cont	other
1	503	459	14	1631	0	0	0	503	473
2	80	230	0	562	0	0	0	80	230
3	131	126	0	389	0	0	0	131	126
4	83	175	0	305	0	0	0	83	175
5	44	23	0	0	0	0	0	44	23
6	15	0	0	4	0	0	0	0	15
7	36	68	0	38	0	0	0	0	104
8	8	6	0	117	0	0	0	0	4
9	0	0	0	243	0	0	0	0	0
10	331	241	0	946	0	0	0	331	241
11	15	0	0	4	0	0	0	0	15
12	120	129	0	403	0	0	0	120	129
13	264	186	52	865	0	0	0	264	238
14	211	112	0	501	0	0	0	211	112
15	143	50	0	265	0	0	0	143	50
16	36	68	0	60	0	0	0	0	104
17	8	6	0	117	0	0	0	0	14
18	103	628	462	396	0	0	0	88	56
19	1166	8880	3770	3618	0	0	0	1322	1066
20	48	197	64	184	0	0	0	33	0
21	1035	9521	1100	4737	0	0	0	1004	483
22	61	600	156	421	0	0	0	77	23
23	139	1296	528	448	0	0	0	136	53
24	692	8272	2719	3137	0	0	0	1021	192
25	0	15	0	10	0	0	0	0	0
26	917	7601	2484	3528	0	0	0	934	252
27	93	554	188	566	0	0	0	103	43
28	546	4743	2306	1811	0	0	0	567	75
29	17	231	39	138	0	0	0	29	5
30	75	353	109	223	0	0	0	55	0
31	15	82	122	43	0	0	0	13	0
32	1020	8290	1825	4596	0	0	0	1030	340
33	252	2613	1689	1063	0	0	0	291	133
34	247	2270	123	1011	0	0	0	242	56
35	0	0	0	0	122	927	0	0	0
36	0	0	0	0	0	10253	1175	0	0
37	0	0	0	0	0	276	0	0	0
38	0	0	0	0	0	9477	692	0	0
39	0	0	0	0	0	717	0	0	0
40	0	0	0	0	0	1720	54	0	0
41	0	0	0	0	0	8154	2316	0	0
42	0	0	0	0	0	15	0	0	0
43	0	0	0	0	20	9140	656	0	0
44	0	0	0	0	0	667	22	0	0
45	0	0	0	0	0	6373	580	0	0

```
   46      0      0      0      0      0    253      0      0      0
   47      0      0      0      0      0    482      0      0      0
   48      0      0      0      0      0    206      0      0      0
   49      0      0      0      0      0   9257    508      0      0
   50      0      0      0      0     20   3586    524      0      0
   51      0      0      0      0     20   2322      0      0      0
 ;
parameter L(m,j,v) load factors;
L(m,j,v) = (tons(m,j)/capacity(v,j))$(capacity(v,j));
parameter a(m) availability of movement m
/
        1      14
        2      14
        3      14
        4      14
        5      15
        6      14
        7      16
        8      16
        9      16
       10      14
       11      14
       12      14
       13      14
       14      14
       15      14
       16      16
       17      16
       18      14
       19      14
       20      24
       21      24
       22      24
       23      24
       24      14
       25      24
       26      24
       27      24
       28      14
       29      14
       30      14
       31      24
       32      24
       33      24
       34      24
       35      14
       36      14
```

```
        37      24
        38      24
        39      24
        40      24
        41      14
        42      24
        43      24
        44      24
        45      14
        46      14
        47      14
        48      24
        49      24
        50      24
        51      24
 /;

parameter rdd(m)
/
         1      18
         2      19
         3      20
         4      21
         5      30
         6      18
         7      18
         8      27
         9      30
        10      17
        11      18
        12      19
        13      20
        14      21
        15      25
        16      18
        17      23
        18      21
        19      24
        20      26
        21      27
        22      29
        23      30
        24      24
        25      26
        26      27
        27      29
        28      24
```

```
        29      27
        30      29
        31      26
        32      27
        33      28
        34      29
        35      24
        36      24
        37      34
        38      34
        39      34
        40      34
        41      24
        42      34
        43      34
        44      34
        45      24
        46      24
        47      24
        48      34
        49      34
        50      34
        51      34
 /;
set e of ORIGINs
/
EAST
GULF
WEST
 /;
set d of DESTINATIONs
/
JAPAN
SKORE
 /;
parameter dest2(m,d) shows which DESTINATION m uses
/
         1.SKORE   = 1
         2.SKORE   = 1
         3.SKORE   = 1
         4.SKORE   = 1
         5.JAPAN   = 1
         6.JAPAN   = 1
         7.JAPAN   = 1
         8.JAPAN   = 1
         9.JAPAN   = 1
        10.SKORE   = 1
```

```
        11.SKORE    = 1
        12.SKORE    = 1
        13.SKORE    = 1
        14.SKORE    = 1
        15.SKORE    = 1
        16.SKORE    = 1
        17.SKORE    = 1
        18.SKORE    = 1
        19.SKORE    = 1
        20.SKORE    = 1
        21.SKORE    = 1
        22.SKORE    = 1
        23.SKORE    = 1
        24.SKORE    = 1
        25.SKORE    = 1
        26.SKORE    = 1
        27.SKORE    = 1
        28.SKORE    = 1
        29.SKORE    = 1
        30.SKORE    = 1
        31.SKORE    = 1
        32.SKORE    = 1
        33.SKORE    = 1
        34.SKORE    = 1
        35.SKORE    = 1
        36.SKORE    = 1
        37.SKORE    = 1
        38.SKORE    = 1
        39.SKORE    = 1
        40.SKORE    = 1
        41.SKORE    = 1
        42.SKORE    = 1
        43.SKORE    = 1
        44.SKORE    = 1
        45.SKORE    = 1
        46.SKORE    = 1
        47.SKORE    = 1
        48.SKORE    = 1
        49.SKORE    = 1
        50.SKORE    = 1
        51.SKORE    = 1
 / ;
parameter origin(m,e) shows which ORIGIN m uses
/
        1.EAST      = 1
        2.EAST      = 1
        3.EAST      = 1
```

```
 4.EAST       = 1
 5.GULF       = 1
 6.WEST       = 1
 7.WEST       = 1
 8.WEST       = 1
 9.WEST       = 1
10.WEST       = 1
11.WEST       = 1
12.WEST       = 1
13.WEST       = 1
14.WEST       = 1
15.WEST       = 1
16.WEST       = 1
17.WEST       = 1
18.EAST       = 1
19.EAST       = 1
20.EAST       = 1
21.EAST       = 1
22.EAST       = 1
23.EAST       = 1
24.GULF       = 1
25.GULF       = 1
26.GULF       = 1
27.GULF       = 1
28.WEST       = 1
29.WEST       = 1
30.WEST       = 1
31.WEST       = 1
32.WEST       = 1
33.WEST       = 1
34.WEST       = 1
35.EAST       = 1
36.EAST       = 1
37.EAST       = 1
38.EAST       = 1
39.EAST       = 1
40.EAST       = 1
41.GULF       = 1
42.GULF       = 1
43.GULF       = 1
44.GULF       = 1
45.WEST       = 1
46.WEST       = 1
47.WEST       = 1
48.WEST       = 1
49.WEST       = 1
50.WEST       = 1
```

```
       51.WEST       = 1
 / ;
parameter b(m,v) latest that m can go on v to meet rdd ;
b(m,'c5')        = rdd(m);
b(m,'c141b')  = rdd(m);
b(m,'c17  ')  = rdd(m);
b(m,'lrwc')    = rdd(m);
b(m,'lrwp')    = rdd(m);
b(m,'bulk')    = rdd(m)-10;
b(m,'cont')    = rdd(m)-10;
b(m,'roro')    = rdd(m)-10;
set     t / n005, n004, n003, n002, n001, c000  * c040 / ;
parameter tnum(t) number of day t;
             tnum(t) = ord(t) - 6;
parameter inuse(e,d,t);
inuse(e,d,t) = sum(m $ ((tnum(t) ge A(m) and tnum(t) le (rdd(m)))
and origin(m,e) and dest2(m,d)), 1);
set     h / n005, n004, n003, n002, n001, c000  * c040 / ;
parameter hnum(h) number of day h;
             hnum(h) = ord(h) - 6;
parameter dollars(v) guesstimate cost of additional lift assets of type v
             /
             c5        0.01
             c141b  0.01
             c17       500
             lrwc    30
             lrwp    0.01
             bulk    0.01
             cont    0.01
             roro    0.01
             / ;
parameter S(e,d,v) cycle time;
S(e,d,'c5') = 2;
S(e,d,'c141b') = 2;
S(e,d,'c17') = 2;
S(e,d,'lrwc') = 2;
S(e,d,'lrwp') = 2;
S(e,d,'bulk') = 20;
S(e,d,'cont') = 20;
S(e,d,'roro') = 20;
parameter n(v) number of lift assets of type v at on hand
             /
             c5        0
             c141b     0
             c17       0
             lrwc      0
             lrwp      0
```

```
        bulk      0
        cont      0
        roro      0
        / ;
```

GAMS Program to Minimize Cost of New Transportation Assets

```
variables
        U(e, d, t, v) no. of v loaded at ORIGIN e in t bound for DESTINATION d
        X(m, j, t, v) no. of v loaded with j of m on day t
        Y(v) number of type v lift assets to be acquired
        z cost to be minimized
;
positive variables X, U, Y;
*positive variable X;
*integer variables U, Y;
equations
        cost define objective function
        cost2 dummy constraint
        demand(m,j) every movement is shipped in its entirety
        supply1(e,d,t,v) supply of shipping
        supply2(h,v) supply of shipping
        lrwc constraint on number of lrwcs
        lrwp constraint on number of lrwps
        c141 constraint on number of c141s
        c5 constraint on number of c5s
        bulk constraint on number of bulks
        cont constraint on number of conts
        roro constraint on number of roros;
cost .. z =e= sum(v, dollars(v)*Y(v)) ;
demand(m,j) $ (tons(m,j)).. sum( (v,t) $
(tnum(t) ge A(m) and tnum(t) le B(m,v) and L(m,j,v)),
  (1/L(m,j,v))*X(m,j,t,v) ) =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
 .. sum( (m,j)$ (origin(m,e) and dest2(m,d) and
(tnum(t) ge A(m) and tnum(t) le B(m,v))), X(m,j,t,v) ) - U(e,d,t,v)  =l= 0 ;
supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
*and inuse(e,d,t)), U(e,d,t,v) )
), U(e,d,t,v) )
                              - Y(v) =l= N(v) ;
lrwc .. Y('lrwc') =l= 15;
lrwp .. Y('lrwp') =l= 75;
c141 .. Y('c141b') =l= 150;
c5 .. Y('c5') =l= 100;
bulk .. Y('bulk') =l=  60;
cont .. Y('cont') =l=  40;
```

```
roro .. Y('roro') =l=  50;
model smmIP / demand, supply1, supply2, cost, lrwc, lrwp, c141, c5,
 bulk, cont, roro /;
option solprint=off, iterlim=100000, reslim=100000 ;
solve smmIP using lp minimizing z ;
display x.l, x.m, y.l, y.m, z.l, z.m;
```

GAMS Program to Minimize "Lateness" Subject to Budget Constraints

```
variables
        U(e, d, t, v) no. of v loaded at ORIGIN e in t bound for DESTINATION d
        X(m, j, t, v) no. of v loaded with j of m on day t
        Y(v) number of type v lift assets to be acquired
        W(m, j, t, v) no. of v loaded with j of m on day t
        z lateness to be minimized

positive variables W, X, U, Y;
*positive variable X;
*integer variables U, Y;
equations
        lateness
        cost define budget constraint
        demand(m,j) every movement is shipped in its entirety
        supply1(e,d,t,v) supply of shipping
        supply2(h,v) supply of shipping
        lrwc constraint on number of lrwcs
        lrwp constraint on number of lrwps
        c141 constraint on number of c141s
        c5 constraint on number of c5s
        bulk constraint on number of bulks
        cont constraint on number of conts
        roro constraint on number of roros;
lateness .. z =e= sum( (m,j,t,v) $
(tnum(t) ge (B(m,v)+1) and tnum(t) le (B(m,v)+9) and tnum(t) gt A(m)),
(tnum(t)-B(m,v))*(capacity(v,j)*W(m,j,t,v))) ;
cost .. sum(v, dollars(v)*Y(v)) =l= 5;
demand(m,j) $ (tons(m,j))..
sum( (v,t) $ (tnum(t) ge A(m) and tnum(t) le B(m,v) and L(m,j,v)),
(1/L(m,j,v))*X(m,j,t,v) ) +
sum( (v,t) $ ( (tnum(t) ge (B(m,v)+1)) and (tnum(t) le (B(m,v)+9)) and
(tnum(t) gt A(m)) and
L(m,j,v)),
(1/L(m,j,v))*W(m,j,t,v) )
 =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
*supply1(e,d,t,v)
```

```
 .. sum( (m,j)$ (origin(m,e) and dest2(m,d) and
(tnum(t) ge A(m) and tnum(t) le B(m,v)+9)),
W(m,j,t,v) +  X(m,j,t,v) ) - U(e,d,t,v)  =l= 0 ;
supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
and inuse(e,d,t)), U(e,d,t,v) )
*), U(e,d,t,v) )
                              - Y(v) =l= N(v) ;
lrwc .. Y('lrwc') =l= 15;
lrwp .. Y('lrwp') =l= 75;
c141 .. Y('c141b') =l= 150;
c5 .. Y('c5') =l= 100;
bulk .. Y('bulk') =l=  60;
cont .. Y('cont') =l=  40;
roro .. Y('roro') =l=  50;
model smmIP / lateness, demand, supply1, supply2, cost,lrwc,
lrwp, c141, c5, bulk, cont, roro /;
option solprint=off, iterlim=100000, reslim=100000 ;
solve smmIP using lp minimizing z ;
display x.l, w.l, y.l, z.l;
```

Early GAMS Program to Minimize "Early" Shipments Subject to Budget Constraints

```
variables
       U(e, d, t, v) no. of v loaded at ORIGIN e in t bound for DESTINATION d
       X(m, j, t, v) no. of v loaded with j of m on day t
       Y(v) number of type v lift assets to be acquired
       W(m, j, t, v) no. of v loaded with j of m on day t
    z earlynes to be minimized
;
positive variables W, X, U, Y;
*positive variable X;
*integer variables U, Y;
equations
       earlynes
       cost define budget constraint
       demand(m,j) every movement is shipped in its entirety
       supply1(e,d,t,v) supply of shipping
       supply2(h,v) supply of shipping
       lrwc constraint on number of lrwcs
       lrwp constraint on number of lrwps
       c141 constraint on number of c141s
       c5 constraint on number of c5s
       bulk constraint on number of bulks
```

```
       cont constraint on number of conts
       roro constraint on number of roros;
earlynes .. z =e= sum( (m,j,t,v) $
(tnum(t) ge (A(m)-8) and tnum(t) le (A(m)-1)),
(A(m)-tnum(t))*(capacity(v,j)*W(m,j,t,v))) ;
cost .. sum(v, dollars(v)*Y(v)) =l= 50000 ;
demand(m,j) $ (tons(m,j))..
sum( (v,t) $ (tnum(t) ge A(m) and tnum(t) le B(m,v) and L(m,j,v)),
(1/L(m,j,v))*X(m,j,t,v) ) +
sum( (v,t) $ ( (tnum(t) ge (A(m)-8)) and (tnum(t) le (A(m)-1)) and
(tnum(t) lt B(m,v)) and
L(m,j,v)),
(1/L(m,j,v))*W(m,j,t,v) )
 =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
*supply1(e,d,t,v)
 .. sum( (m,j)$ (origin(m,e) and dest2(m,d) and
(tnum(t) ge A(m)-8 and tnum(t) le B(m,v)) ),
W(m,j,t,v) +  X(m,j,t,v) ) - U(e,d,t,v)  =l= 0 ;

supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
and inuse(e,d,t)), U(e,d,t,v) )
*), U(e,d,t,v) )
                              - Y(v) =l= N(v) ;
lrwc .. Y('lrwc') =l= 15;
lrwp .. Y('lrwp') =l= 75;
c141 .. Y('c141b') =l= 150;
c5 .. Y('c5') =l= 100;
bulk .. Y('bulk') =l=  60;
cont .. Y('cont') =l=  40;
roro .. Y('roro') =l=  50;
model smmIP / earlynes, demand, supply1, supply2, cost,lrwc,
lrwp, c141, c5, bulk, cont, roro /;
option solprint=off, iterlim=100000, reslim=100000 ;
solve smmIP using lp minimizing z ;
display x.l, w.l, y.l, z.l;
```

GAMS Program to Minimize Cargo Prepositioned Subject to Budget Constraints

```
variables
       U(e, d, t, v) no. of v loaded at ORIGIN e in t bound for DESTINATION d
       X(m, j, t, v) no. of v loaded with j of m on day t
       Y(v) number of type v lift assets to be acquired
   W(m, j, v) no. of v loaded with j of m on day t
   z prepo to be minimized
```

```
positive variables W, X, U, Y;
*positive variable X;
*integer variables U, Y;
equations
      prepo
      cost define budget constraint
      demand(m,j) every movement is shipped in its entirety
      supply1(e,d,t,v) supply of shipping
      supply2(h,v) supply of shipping
      lrwc constraint on number of lrwcs
      lrwp constraint on number of lrwps
      c141 constraint on number of c141s
      c5 constraint on number of c5s
      bulk constraint on number of bulks
      cont constraint on number of conts
      roro constraint on number of roros;
prepo .. z =e= sum( (m,j,v),
capacity(v,j)*W(m,j,v) ) ;
cost .. sum(v, dollars(v)*Y(v)) =l= 5 ;
demand(m,j) $ (tons(m,j))..
sum( (v,t) $ (tnum(t) ge A(m) and tnum(t) le B(m,v) and L(m,j,v)),
(1/L(m,j,v))*X(m,j,t,v) ) +
sum( (v) $ L(m,j,v),
(1/L(m,j,v))*W(m,j,v) )
 =e= 1 ;
supply1(e,d,t,v)$(inuse(e,d,t))
 .. sum( (m,j)$ (origin(m,e) and dest2(m,d) and
(tnum(t) ge A(m) and tnum(t) le B(m,v)) ),
X(m,j,t,v) ) - U(e,d,t,v)  =l= 0 ;
supply2(h,v) .. sum( (e,d,t) $
( (tnum(t) ge (hnum(h)-S(e,d,v))) and (tnum(t) le hnum(h))
and inuse(e,d,t)), U(e,d,t,v) )
                           - Y(v) =l= N(v) ;
lrwc .. Y('lrwc') =l= 15;
lrwp .. Y('lrwp') =l= 75;
c141 .. Y('c141b') =l= 150;
c5 .. Y('c5') =l= 100;
bulk .. Y('bulk') =l=  60;
cont .. Y('cont') =l=  40;
roro .. Y('roro') =l=  50;
model smmIP / prepo, demand, supply1, supply2, cost,lrwc,
lrwp, c141, c5, bulk, cont, roro /;
option solprint=off, iterlim=100000, reslim=100000 ;
solve smmIP using lp minimizing z ;
display x.l, w.l, y.l, z.l;
```

Appendix B
SENSITIVITY ANALYSIS

The sensitivity analysis merits a more detailed discussion. Table 3.11 presents the shadow prices and allowable ranges for the 14 Equation 3.6 constraints in this example—ten movement requirements, four of them with two cargo types. Movements with shadow prices that are not zero indicate some potential for savings. Note, for example, that there would be no change in the optimal value of the objective function in this relaxed solution if prepositioning or early arrival of movement requirements 1 through 5 or 10 was achieved. In fact, they could be increased in size without affecting the value of the objective function; for example, the first movement could increase to 1.73 times its size before affecting the optimum. Actually, some reduction in the objective function *might* be achieved for decreases beyond the allowable range; for example, if more than 25 percent of the movement 5 personnel could be prepositioned, some savings *might* accrue.

PREPOSITIONING

Of far greater interest are movements 6 through 9, because their shadow prices suggest potential for savings. The largest payoff would appear to be movement 8; if 55 percent of it (0.55 from the "Allowable Decrease" column of Table 3.11) could be prepositioned, then the objective function (from Table 3.6) would be decreased to 7.42 – 7.663(0.55). Whether this decrease would translate into the saving of a lift asset cannot be immediately ascertained, but the parametric analysis of the right-hand side of the constraint that describes this movement can provide a great deal of insight.

Table B.1 presents a portion of this parametric analysis for movement requirement 8. The first line describes the optimal solution: If all values are rounded to two or three decimal places, the right-hand side is equal to 1.00 (all the movement requirement must be shipped), the objective function is equal to 7.42, and the shadow price is –7.663. The second line shows the effect of prepositioning or early arrival of up to 0.55 of the movement (so that between 0.45 and 1.00 must be shipped within the stated time window); if 0.055 of the 710 tons (approximately 35.5 tons) could be prepositioned, the objective function would be reduced to

$$7.42 - 7.663(0.055) = 7.00$$

Table B.1

Parametric Analysis of Movement Requirement 8

Variable Out	Variable In	Constraint Value	Shadow Price	Objective Value
—	—	1.000	−7.66	7.42
X(8, Bulk, 9, KC-10)	X(9, Bulk, 10, KC-10)	0.454	−7.66	3.24
X(8, Bulk, 10, KC-10)	U(New York, Tianan, 7, KC-10)	0.454	−7.66	3.24
X(8, Bulk, 8, KC-10)	X(6, Bulk, 7, C-5)	0.419	−7.65	2.97
Slack	X(10, Over, 12, KC-10)	0.417	−7.65	2.95
Slack	X(7, Bulk, 6, KC-10)	0.417	−7.65	2.95
.	.	.	.	.

NOTE: Dashes indicate original value, no variable.

However, this is not a real integer saving even though 7.00 is a feasible integer value of the objective function—the cost of three KC-10s and one C-141.

To gain a saving in aircraft, the integer must be reduced to 6.00 (the cost of 3 KC-10s). The reduction in the objective function comes about by reducing the optimal number of KC-10s from 3.71 to 3.5 in the linear solution (see Table 3.6). Reducing the 710 bulk tons of movement 8 by 0.185, or 131.35 bulk tons, reduces the objective function to

$$7.42 - 7.663(0.185) = 6.00 \ ,$$

the cost of three KC-10s. Note again that only KC-10s are being affected and that this solution would carry over to the integer case: If 131.35 of the 710 bulk tons of movement requirement 8 could be prepositioned, the remainder of this movement plus all the other nine movements could be delivered within their time windows, using only three KC-10s rather than the four required if no prepositioning was done.

What if none of movement 8 is available for prepositioning but certain of the other movement requirements are? Clearly, if these movements have zero shadow prices, they are not candidates for prepositioning. Further, those with small shadow prices may also offer no real benefits; the bulk portion of movement 6, for example, even if totally prepositioned, would reduce the objective function by only 0.225, to 7.194, which would still require four KC-10s. The only other viable candidate for prepositioning is movement 9, but a parametric analysis would reveal that even if the entire movement were prepositioned, the objective function would not fall below 6.89 and four KC-10s would still be required.

In a more realistic, larger problem, a number of movements would have to be found whose prepositioning would result in lift asset acquisition savings. These sensitivity analyses can be performed using only one movement at a time. The simultaneous prepositioning of more than one movement may not result in savings equal to the sum of the savings promised by the shadow prices.

EARLY ARRIVAL

This same kind of analysis can be used to examine early arrival rather than prepositioning. Suppose that 0.185 of movement 8, whose prepositioning could reduce the objective function to 6.00, is not prepositioned but could be at the POE a day or two early. Would this possibility also result in the saving of one lift asset? The answer is, It depends. Having this cargo available on a day when insufficient lift is available to handle it would not provide any benefit.

Suppose that a portion of movement 8 was available two days earlier, on day 5. The constraints that determine the number of lift assets available on day 5 (movements 4 and 5) indicate that there is sufficient slack (shadow price of zero) in the system to allow loading on day 5 and realize the same savings that prepositioning would bring. The constraints for day 7 show no slack (a shadow price of other than zero), hence availability on day 6 may not result in real savings, because the round-trip time of two days means the extra assets used on day 6 would not be available on day 7.

As with prepositioning, the only meaningful candidates for early arrival are those movement requirements whose constraints show shadow prices other than zero. Chapter Five shows a more direct method for examining the benefits of earlier availability.

REFERENCES

Cross, MAJ Stephen, *A Proposed Initiative in Crisis Action Planning,* Defense Advanced Research Projects Agency, Arlington, Va., 1990.

Dantzig, George Bernard, *Notes on Linear Programming—Part XV: Minimizing the Number of Carriers to Meet a Fixed Schedule,* Santa Monica, Calif., RAND, RM-1328, August 1954.

Ferguson, Allen R., *Notes on Linear Programming—Part XVI: The Problem of Routing Aircraft—A Mathematical Solution,* Santa Monica, Calif., RAND, RM-1369, September 1954.

Ferguson, Allen R., *Notes on Linear Programming—Part XXXVI: The Allocation of Aircraft to Routes—An Example of Linear Programming Under Uncertain Demand,* Santa Monica, Calif., RAND, RM-1833, December 1956.

Jarvis, J. J., and H. D. Ratliff, *Models and Concepts: System for Closure Optimization Planning and Evaluation (SCOPE),* Document PRDC-83-06, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, 1983.

Moore, S. C., J. Stucker, and J. Schank, *Wartime Roles and Capabilities for the Unified Logistic Staffs,* Santa Monica, Calif., RAND, R-3716-JCS, February 1989.

Schank, J., J. Stucker, G. Sumner, and M. Mattock, *Enhancing Joint Capabilities in Theater Ammunition Management,* Santa Monica, Calif., RAND, R-3789-JS, 1991a.

Schank, J., et al., *A Review of Strategic Mobility Models and Analysis,* Santa Monica, Calif., RAND, R-3926, 1991b.